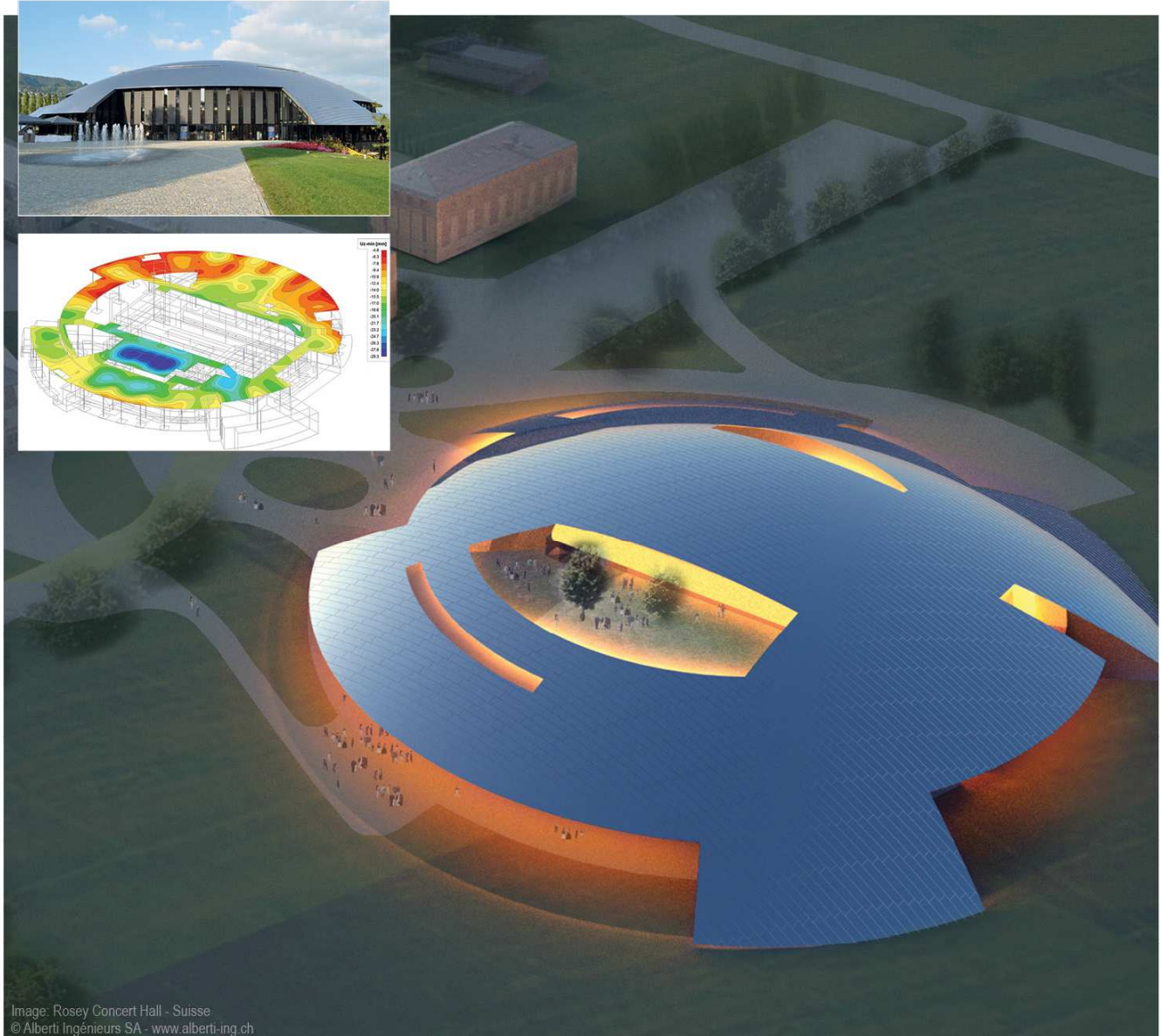


SCIENGINEER



Advanced Expert Training Post-tensioning

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Introduction

All discussed topics are available in the **Expert Edition** or the **Precast Edition** of SCIA Engineer.

The precast module of SCIA Engineer allows the making of an analysis of pre-tensioned or post-tensioned structures. The difference between calculating a pre-tensioned or post-tensioned construction is only in the input. With a pre-tensioned construction, one works with a bore hole pattern. With post-tensioned structures, one has to determine the path of the cable. This course document will treat some examples of post-tensioned constructions.

When making a linear analysis of a post-tensioned construction, there is no limitation concerning the model, which is definitely a strong point of the software. In a 'General XYZ' environment one can perfectly model and analyze the most complex post-tensioned constructions. Here analyzing is meant as a linear calculation with the finite element method of a post-tensioned construction.

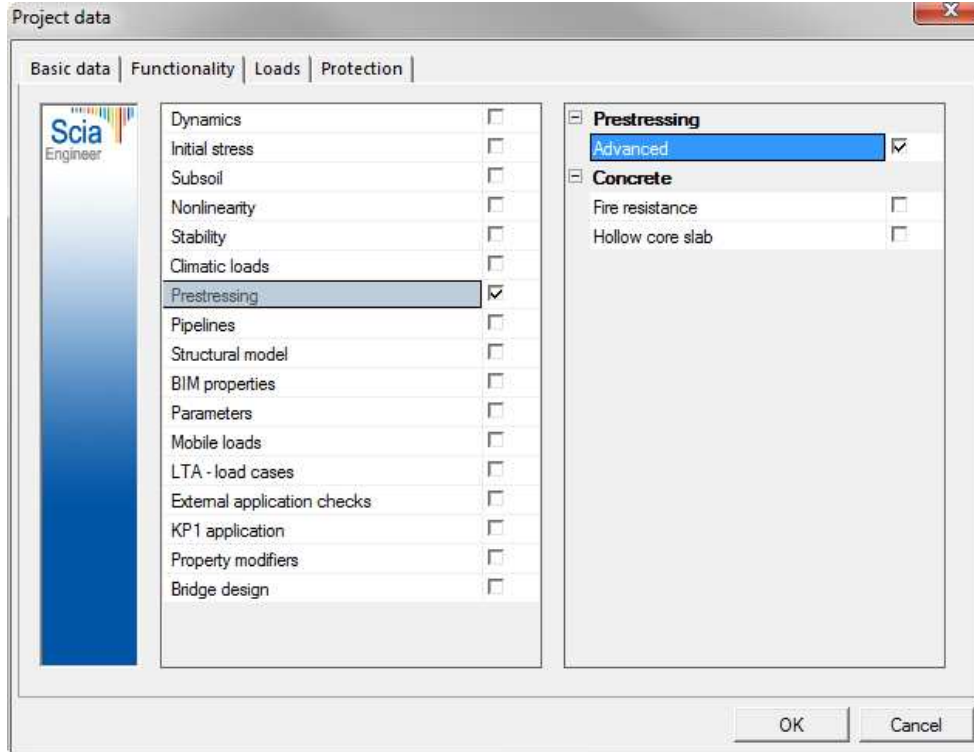
The limitation is that long term losses can't be calculated, because a time dependent analysis is only possible in a 2D environment (XZ frame). This is why we shall limit the first two examples to a 2D-environment, to see how these time dependant losses can be calculated. Moreover, one can also execute the necessary Euro Code controls on a post-tensioned beam.

The first part shall cover an example on a very simple rectangular beam en has the sole purpose to explain the input en output possibilities of a post-tensioned Construction.
The second part will cover a U-formed bridge (Dutch: 'trogligger') where the post-tensioned cable is inputted by means of source geometry. Next there will be a time-dependant analysis executed on it to calculate shrink, creep and relaxation by using the TDA module (time-dependant analysis).
The third and fourth chapter will contain examples about a post-tensioned plate model in the 'General XYZ' environment.

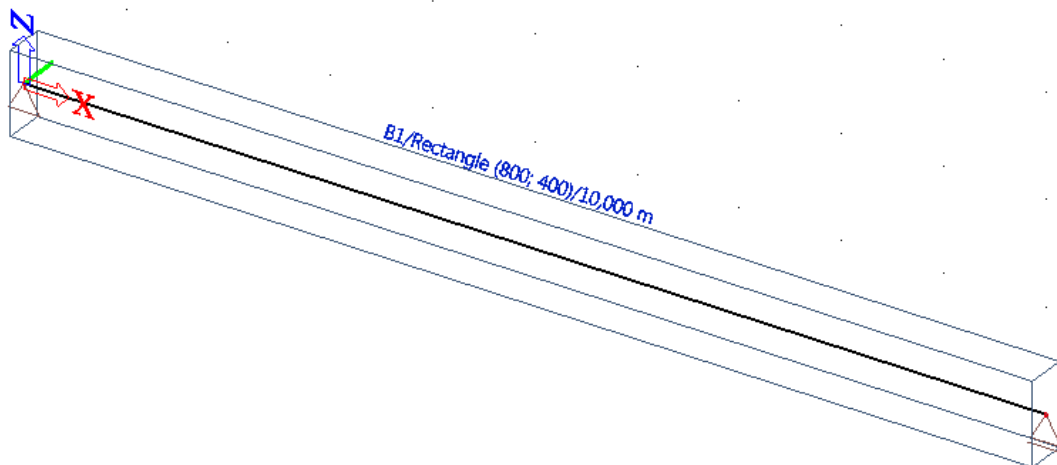
Linear analysis of a post-tensioned beam

1_Input of geometry and post-tensioning

- Before starting a project with pre- or post-tensioning, the 'Prestress' option must be checked in the functionalities menu.



- Next the beam can be modelled. In this example, we will use a rectangular beam (400mm by 800mm, in concrete quality C60/75) of 10m length, which is carried by hinged supports at both ends.



- Adding the post-tensioning is done in the structure menu. Under 'tendons' you will find the possibility to input 'Post-tensioned tendons'. (Remark: inputting pre-tensioned tendons is done in the concrete menu)

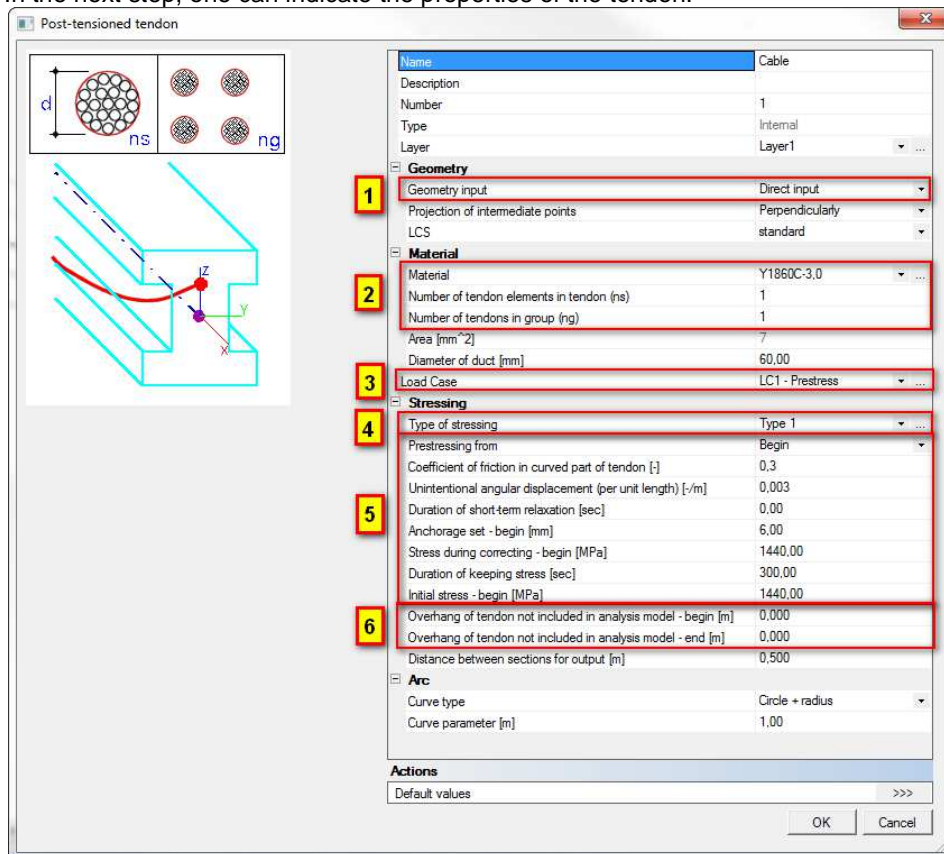
The input of post-tensioned tendons can be done in three ways:

- Through direct input, where the user insert the cable himself into the model.
- Through direct input, where the user imports the cable path through dwg/dxf and converts it into a cable.
- Through source geometry where the user inputs coordinates for the cable and the program will interpolate between these point through possible geometric methods (which the user also indicates).

None of these three options will calculate the optimal cable path. The responsibility for the input lies completely with the user. The software will do the analysis after the input. Most users use option 2 for the input of the cable line. The dwg/dxf usually comes from a graphical design program (like AutoCad, ...).

In the first example we start with the most simple method and insert to straight cables by using option 1.

- Before a tendon can be inputted, a load case of the type 'Prestress' is made. This is needed to store the loads that are caused by the tensioning. Adding tensioning to a structure is seen as a (favourable) load on the model. By making multiple load cases of the type 'prestress', one can model the sequentionnaly tensioning of the cables and the complementary losses. The sequentional post-tensioning can also be modelled with linear construction stages. The load cases 'prestress' will have to be added to each construction phase.
- In the next step, one can indicate the properties of the tendon:



Part 1. This indicates how you will input the tendon.

Part 2. What is the choice of material and number of tendon elements for each tendon.

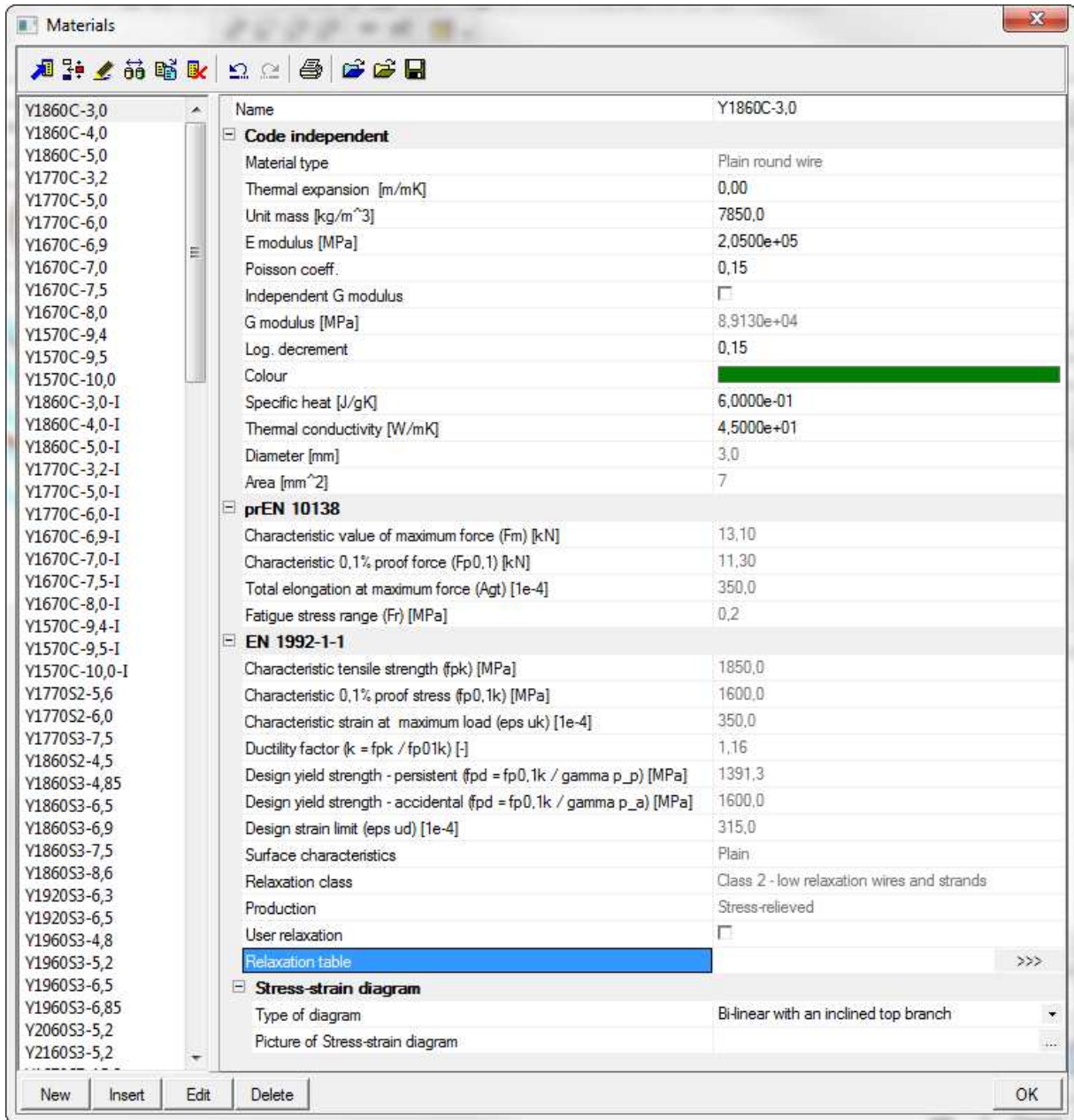
Part 3. To which load case of type 'Prestress' will the tendon be added.

Part 4. The type of prestressing.

Part 5. What are the parameters for the specified type of prestressing.

Part 6. To model a certain amount of overhang.

- Part 2: The choice of material:

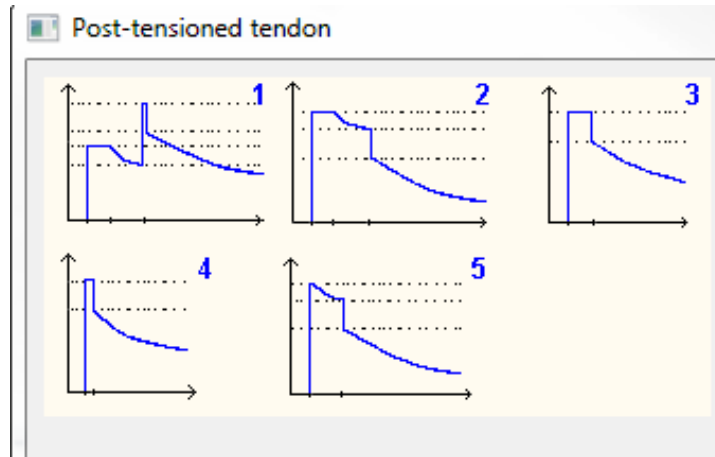


The material properties also contain the diameter of the pretensioning material. In the properties of the tendon itself, you can only change the number of the principal tendon.

The relaxation table can also be reviewed in the material properties. It is copied from the code, but can also be inserted or changed by the user himself. The relaxation properties will obviously only have an influence on the time dependant analysis.

In this example, we chose the material **Y1860C -5,0** with **20 tendons** in one principle tendon and **4** principle tendons in one tendon group.

- **Part 4:** The type of stressing:



The type chosen depends on the way the tendon will be post-tensioned.

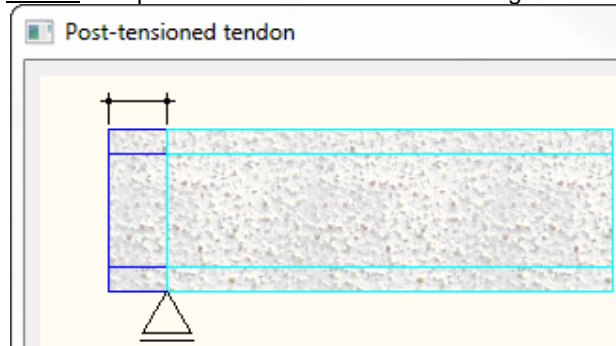
The indicated diagram will be used to calculate immediate losses. For all 5 types there is anchorage set loss (in type 4, this is the only immediate loss).

In the types 1, 2, 3 and 5, there are also other losses, such as short term relaxation, etc ...

The long term losses, as shown in the 5 types, can only be determined by time dependant analysis.

In this example we will use type 4.

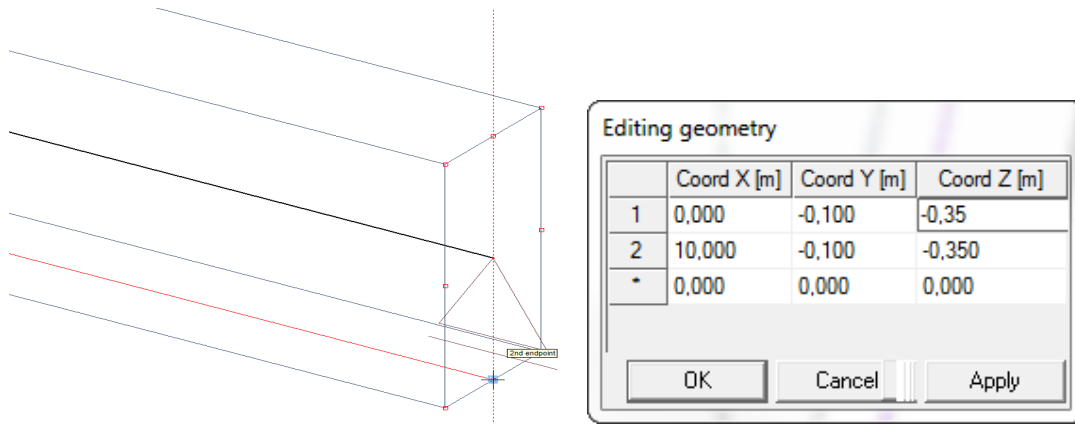
- **Part 5:** The other parameters are used to further define the type of post-tensioning. This is all indicated as a property of the tendon.
So for example, the amount of losses due to anchorage set loss is inputted here. Also the initial stress in the tendon is a very important input value. In this example it is taken equal to 1440 MPa.
With post-tensioned cables, the loss due to friction is also taken into account. So therefore it becomes important whether the cable is stressed at the beginning or end.
- **Part 6:** The possibilities to define the overhang can be seen in the following image:



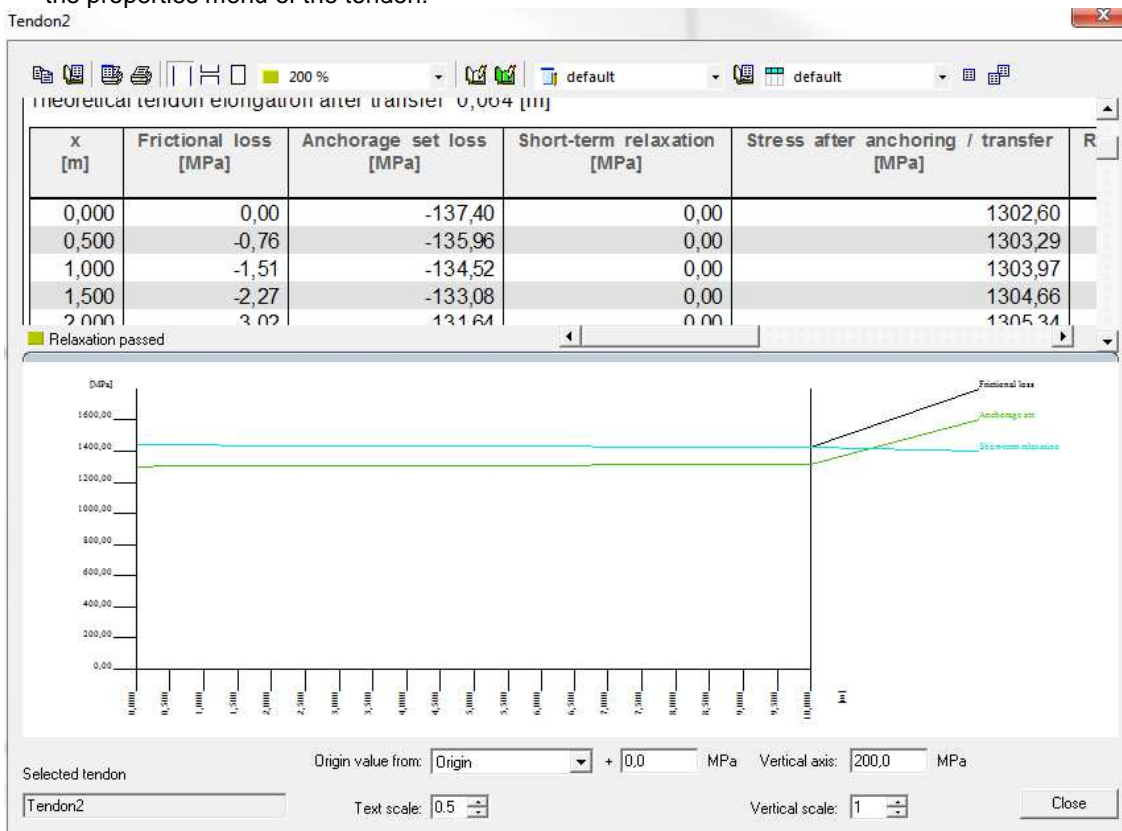
This overhang is not taken into account for the FEM analysis, but it is used for the calculation of the losses in the cable.

The next step is inserting the cable. Even after inserting the cable, it will still be possible to review and change the properties set in the previous steps.

The user can use the 'snap settings' to place the cable and afterwards use more precision for the placement of the tendon by using the 'Table edit geometry' button (for the action buttons in the properties of the tendon).



- Even before the analysis is executed, it is possible to look at the immediate losses in the stressing of the tendon. To see these, the user can click on 'Tendon losses' between the action buttons in the properties menu of the tendon:



If tensioning is done at the beginning of a tendon, then there is a big anchorage set loss which becomes smaller towards the end. The loss due to friction increases towards the end of the cable.

Each 0,5m (which is configured in the properties of the tendon), the stress after transfer is determined. This is the stress found after subtraction of anchorage set loss and the friction loss from the initial stress.

This stress at transfer (SAT) is the stress which is sent to the solver to execute the analysis.

- Similar to the input of the first tendon, we will also add a second tendon in the other bottom corner. This can be simply copied from the original tendon. And after which you will have to assign the second cable to a certain load case (now a different one due to construction stages).

2_Construction stages (and loads)

Three linear construction stages will be made.

Stage 1: Only 'Self weight'

Stage 2: Tensioning tendon 1 in 'LC 1 – Prestress cable 1' (with an additional empty load case)

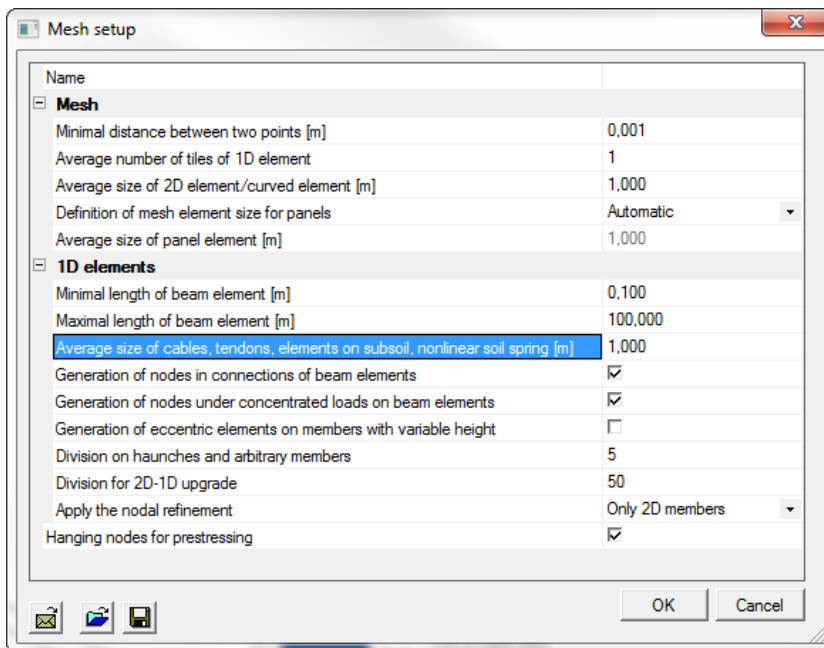
Stage 3: Tensioning tendon 2 in 'LC 2 – Prestress cable 2' (with an additional empty load case)

In this simple example, time dependant analysis will not be used. There will also not be any other stage which includes service loads.

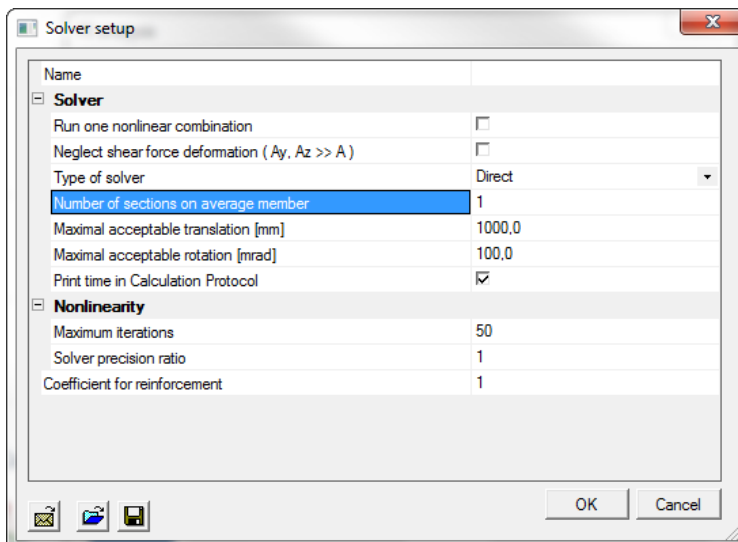
3_Calculation

Before a calculation with prestressed tendons can be performed, the mesh and solver setups will have to be modified.

Mesh: It is very important that the tendons are divided in sufficient finite elements to ensure an accurate analysis.



Solver: If the size of the mesh is sufficiently small, then the number of sections on average member can be set to 1. This will benefit the speed of postprocessors, such as Euro Code checks.



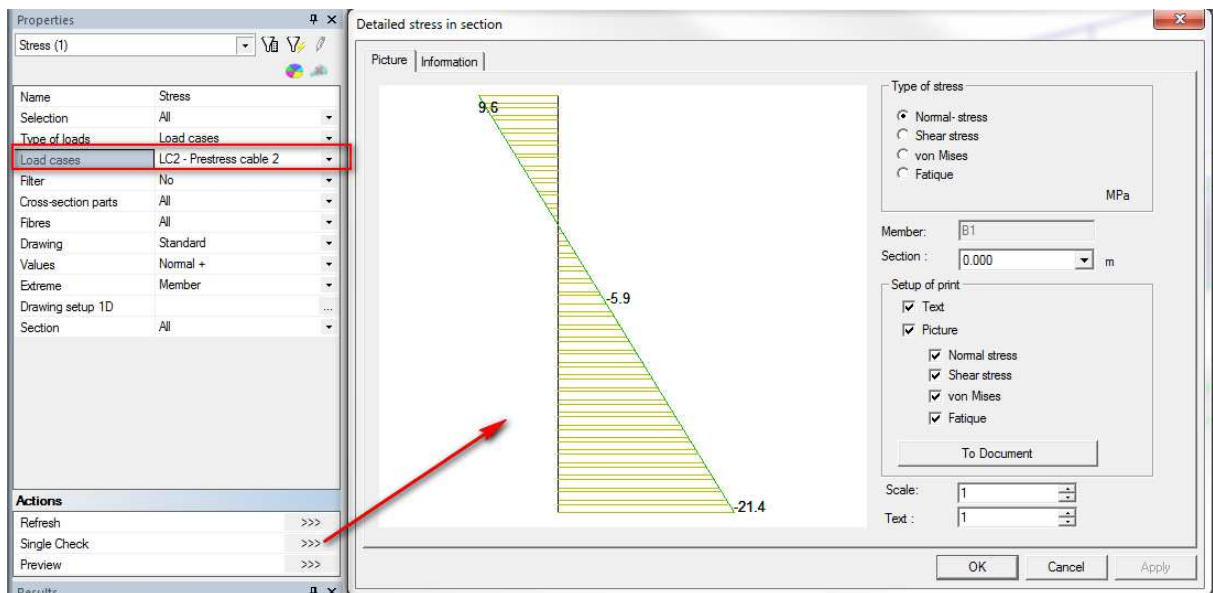
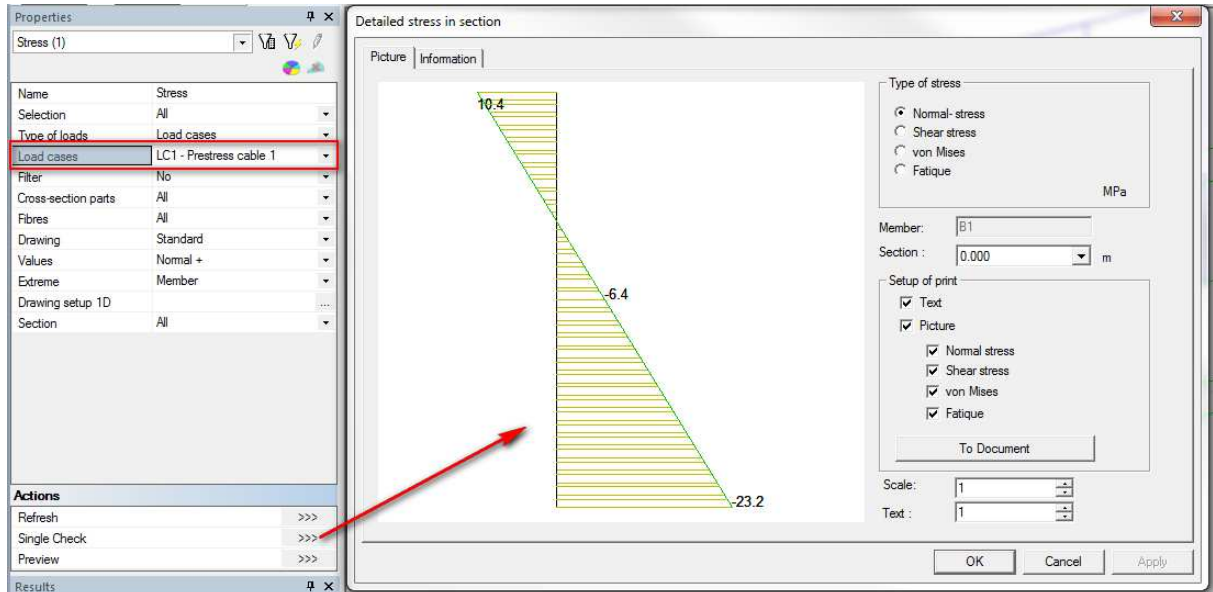
4_Results

In the results menu the 'Internal forces' and 'Tendon Stresses' can be viewed. Also for each individual load case, as for each class that is made for a construction stage, the results can be viewed.

We will compare the results from 'LC1 – Prestress cable 1' with 'LC2 – Prestress cable 2':

→ In the stresses on beam, there is a slightly bigger stress for 'LC1', then for 'LC 2'.

→ This is logical considering that cable 2 will lose some tendon stress since cable 1 will be the first to be tensioned. The same result will also become clear in the tendon stresses.



If we would consider the normal forces (under internal forces), then we would see that these are lower for LC2 than for LC1.

Note: There is an option 'Prestress' in the property menu of 'Internal forces'. If this is ticked on, then the section will be calculated as possibly already prestressed. For LC1 this will not make a difference, but for LC2 this can give different results, because at the time of tensioning for cable 2, there is already stress in the section due to cable 1. This will make the equivalent section (and the moment of inertia) bigger, which causes the results for LC2 not to be the same as those of LC1 for a regular section.

Remark: If the option 'Prestress' is ticked on, then it will be possible to chose for 'Total resultants' or just 'Primary forces' or 'Secondary forces'. In an isostatic beam, the secondary forces will be zero, because these only appear with isostatic structure (example 5). A background about these secondary forces is given at p86 of the book 'Navrátil, J.: Prestressed concrete structures'.

The tendon stresses shows the course of the stresses over the length of the tendon.

- **SAT** = *Stress of transfer*: This is the initial stress, reduced by the immediate losses.
- **LED** (= *Loss due to elastic deformation*): These losses are due to sequential post-tensioning and elastic deformation of the concrete. These losses will only appear in stage 3 (tensioning 2nd cable).
- **LCS** (= *Loss due to creep & shrinkage*): These are the long term losses due to creep, shrinkage and long term relaxation. They are only calculated in a time dependent analysis (and so they are now 0).

The preview of these losses can be seen in the image below.

Tendon stresses

Tendon stresses

Linear calculation, Extreme : Global

Selection : All

Tendons: All by selection

Class : ST3 (ULS)

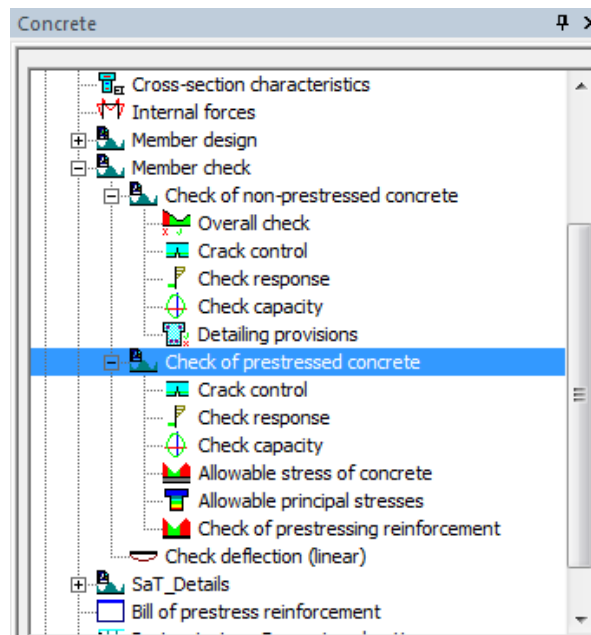
Case	Tendon	x [m]	Stress after anchoring / transfer [MPa]	LED [MPa]	LCS [MPa]	Min Stress [MPa]	Max Stress [MPa]
ST3 (ULS)	Tendon1	0,000	1302,60	-101,88	0,00	1200,72	1200,72
ST3 (ULS)	Tendon1	10,000	1316,35	-102,96	0,00	1213,40	1213,40

5_Controls in the concrete menu and design of passive reinforcement

In the concrete menu you can check if the post-tensioned beam will suffice. This concerns the design of passive reinforcement, as well as the specific Euro Code checks for the prestress steel.

The steps to execute the design of the passive reinforcement and to use the complementary checks (crack control, response control and capacity control) are completely consistent with the design of reinforcement of 1D elements. This is why a reference will be made to the workshop concrete.

The Euro Code checks which are specific for prestress elements are the check of 'Allowable stress of concrete', 'Allowable principal stress' and the 'Check of prestressing reinforcement'.



Check of the 'Allowable stress of concrete

There are multiple checks (three) to perform for the allowable stress of concrete.

- Stage 3 → *Ultimate Limit State combination (ULS): $\sigma_{cc,max}$*

The value $\sigma_{cc,aa}$ will be checked (maximum stress of concrete in compression after anchorage). This stress should be smaller than $\sigma_{cc,max}$ (maximum allowable stress of the concrete) in order for the concrete not to be crushed. The value $\sigma_{cc,aa}$ obviously exceeds $\sigma_{cc,max}$.

Properties: Allowable stress concrete EN 1992-1-1 (1)

Preview: Allowable stress concrete EN 1992-1-1

Linear calculation, Extreme : Member
Selection : All
Combinations : F3-EN-ULS (STR/GEO) Set B

Prestress check of allowable stress concrete for selected members

Member	d _s [m]	Case	Fibre	N [kN]	M _y [kNm]	M _z [kNm]	$\sigma_{cc,aa}$ [MPa]	$\sigma_{cc,max}$ [MPa]
B1	0,000	F3-EN-ULS (STR/GEO) Set B/2	7	-4913,2	-1719,6	0,0	-54,94	-36,00

- Stage 3 → *Characteristic Service Limit State combination (SLS): $f_{ct,eff}$*

The value $\sigma_{cq,max}$ (maximum concrete stress after application of self-weight and all loads) must not exceed the allowable tension in the concrete, $f_{ct,eff}$ in an SLS combination.

This value is strongly exceeded when all loads are applied because in this example, the upper fibre of the beam is subjected to tension (due to the prestressed tendons). In a practical case, there will be more loads present, and thus the value of $\sigma_{cq,max}$ will be much smaller.

Properties: Allowable stress concrete EN 1992-1-1

Preview: Allowable stress concrete EN 1992-1-1

Linear calculation, Extreme : Member
Selection : All
Combinations : F3-EN-SLS Characteristic

Prestress check of allowable stress concrete for selected members

Member	d _s [m]	Case	Fibre	N [kN]	M _y [kNm]	M _z [kNm]	$\sigma_{c,aa}$ [MPa]	$\sigma_{c,aa,max}$ [MPa]	$\sigma_{c,aa,min}$ [MPa]	$\sigma_{c,aa,max}$ [MPa]	$f_{ct,eff}$ [MPa]
B1	0,000	F3-EN-SLS Characteristic/3	7	-4094,3	-1433,0	0,0	-54,94	-60,43	0,00	-14,25	4,40

Remark: The value $\sigma_{cc,ch}$ (allowable compression in concrete in SLS characteristic combinations) is not calculated because this check is only needed for the environmental classes XD, XF and XS.

- Stage 3 → *Quasi permanent Service Limit State Combination (SLS): $\sigma_{cc,qp}$ and $f_{ct,eff,qp}$*

Here the values of $\sigma_{cc,qp}$ and $f_{ct,eff,qp}$ (respectively the allowable compressive and tensile concrete stress in SLS quasi permanent combinations) are being checked. The values of $\sigma_{lt,min}$ and $\sigma_{lt,max}$ (respectively the minimal and maximal concrete stress caused by long term loads) may not exceed the allowable stresses indicated by $\sigma_{cc,qp}$ and $f_{ct,eff,qp}$. The user can modify $f_{ct,eff,qp}$ manually in the concrete setup. By default it is set to zero. So since there is tension in some of the fibres, the check will not pass.

Allowable stress concrete EN 1992-1-1

Linear calculation, Extreme : Member
 Selection : All
 Combinations : F3-EN-SLS Quasi-permanent

Prestress check of allowable stress concrete for selected members

Member	d_s [m]	Case	Fibre	N [kN]	M_{ed} [kNm]	σ_{pa} [MPa]	σ_{pa} [MPa]	σ_{pm} [MPa]	σ_{pm} [MPa]	σ_{pm} [MPa]	σ_{pm} [MPa]	σ_{pm} [MPa]	σ_{pm} [MPa]
B1	10,000	F3-EN-SLS Quasi-permanent/3	3	-4133,4	-1446,7	-55,46	-55,46	23,49	23,49	0,00	0,00	-61,01	25,84

Check of allowable principal stresses

This check is described in the Euro Code EN1992, art 12.6.3 (3):

<< A concrete member may be considered to be uncracked in the ultimate limit state if either it remains completely under compression or if the absolute value of the principal concrete tensile stress σ_{ct1} does not exceed f_{ctd} . >>

In the check of allowable principal stresses it can be seen that this check is not alright:

Prestress check allowable principal stresses EN 1992-1-1

Linear calculation, Extreme : Member
 Selection : All
 Combinations : F3-EN-ULS (STR/GEO) Set B

Prestress check of allowable principal stresses for selected members

Member	d_s [m]	Case	N [kN]	V_{ed} [kN]	V_{zd} [kN]	M_{kd} [kNm]	M_{rd} [kNm]	M_{rd} [kNm]	σ_{ct1} [MPa]	f_{ctd} [MPa]
B1	10,000	F3-EN-ULS (STR/GEO) Set B/2	-4960,1	0,0	-53,0	0,0	-1736,0	0,0	26,89	2,07

Check of stress in the prestressing reinforcement

This option is used to check the stress in the post-tensioned tendons for an ULS combination.

Combinations : F3-EN-ULS (STR/GEO) Set B

Check of prestressing reinforcement for selected tendons

Tendon	d_s [m]	Case	σ_{pa} [MPa]	σ_{pa} [MPa]	σ_{pm} [MPa]	σ_{pm} [MPa]	σ_{pm} [MPa]	σ_{pm} [MPa]	Check _{cal} [-]	Check _{lim} [-]	Check
Tendon1	0,00	F3-EN-ULS (STR/GEO) Set B/2	1440,00	1302,60	1180,35	1180,35	1180,35	1180,35	1,00	1,00	OK
		ST3	1440,00	1360,00	1395,00	1395,00	1395,00	1395,00	1,00	1,00	

These checks pass. No stress is exceeded. In the image below there is more explanation about the symbols used in the preview. This way it becomes easier to manually see what is checked.

$\sigma_{p,pa}$	Stress of prestressing reinforcement prior anchoring (during tensioning)
$\sigma_{p,aa}$	Stress of prestressing reinforcement after anchoring/transfer of prestress
$\sigma_{pq,min}$	Minimum stress of prestressing reinforcement after application of self-weight, all permanent, and variable loads
$\sigma_{pq,max}$	Maximum stress of prestressing reinforcement after application of self-weight, all permanent, and variable loads
$\sigma_{p,ltl}$	Stress of prestressing reinforcement after long-term losses (LTL)
Check _{cal}	Maximum value of check of all performed checks of allowable stress of tendon
$\sigma_{p,max}$	Allowable stress of prestressing reinforcement prior anchoring (during tensioning)
σ_{pm0}	Allowable stress of prestressing reinforcement after anchoring/transfer of prestress
σ_{pm}	Allowable stress of prestressing reinforcement caused by SLS combinations

The allowable stresses before and after anchorage ($\sigma_{p,max}$ and $\sigma_{p,m0}$) are calculated according to chapter 5.10.2 and 5.10.3 of the Euro Code EN1992. The allowable stress $\sigma_{p,m}$ in the tendons under the service limit state combinations are given in chapter 7.2.

The factors used in these formula can be configured in the national annex for Euro Code 2, in the part which indicates national dependent values for EN1992-1-1. The parameters can be found under the part 'Allowable stress'.

Name	Standard EN
Concrete	
General	
ULS	
SLS	
Allowable stress	
Stress limitation during tensioning	
National annex	
k1: k2 to calculate the maximum stress applied to the tendon during tensioning	
Values [-]	0,80 / 0,90
k3 - increased factor for maximum stress in prestressing reinforcement during tensioning 5.10.2.1(2)	
Value [-]	0,95
k6 - increased factor for maximum compressive stress in pre-tensioned concrete after transfer of prestress 5.10.2.2(5)	
Value [-]	0,70
k7 - factor for maximum stress in prestressing reinforcement after anchoring/transfer of prestress 5.10.3 (2)	
Value [-]	0,75
k8 - factor for maximum stress in prestressing reinforcement after anchoring/transfer of prestress 5.10.3 (2)	
Value [-]	0,85
SLS stress limitation	
National annex	
k1 - factor for maximum compressive stress in concrete under SLS characteristic combi	
Value [-]	0,60
k2 - factor for maximum compressive stress in concrete under SLS quasi-permanent combi (linearity of creep condition) 7.2(3)	
Value [-]	0,45
k5 - factor for maximum stress in prestressing reinforcement under SLS combi 7.2(5)	
Value [-]	0,75
Detailing provisions	
Common detailing provisions	
Columns	
Beams	

The stresses indicated in the preview of this check can easily be calculated manually:

➔ 5.10.2 Maximum Stressing Force

- $k_1 = 0,80$
- $k_2 = 0,90$
- $\sigma_{p,max} = \min(k_1 \cdot f_{pk}; k_2 \cdot f_{p,01k}) = 1440 \text{ N/mm}^2$

➔ 5.10.3 Prestressing Force

- $k_7 = 0,75$
- $k_8 = 0,85$
- $\sigma_{p,m0} = \min(k_7 \cdot f_{pk}; k_8 \cdot f_{p,01k}) = 1360 \text{ N/mm}^2$

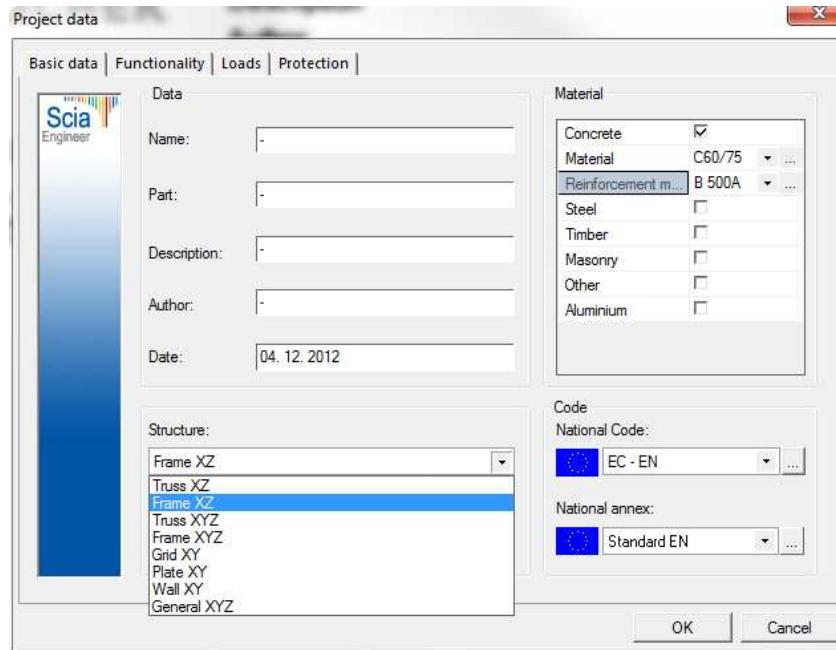
Summary

The checks concerning allowable stresses in the concrete were not all right, but the checks for allowable stress in the prestress tendons did pass.

Now the user will have to modify his design to ensure that the check for allowable concrete stresses pass the unity check. It can be done by increasing the size of the cross section or by augmenting the quality of the concrete used in the beam.

Time dependant analysis of a post-tensioned beam

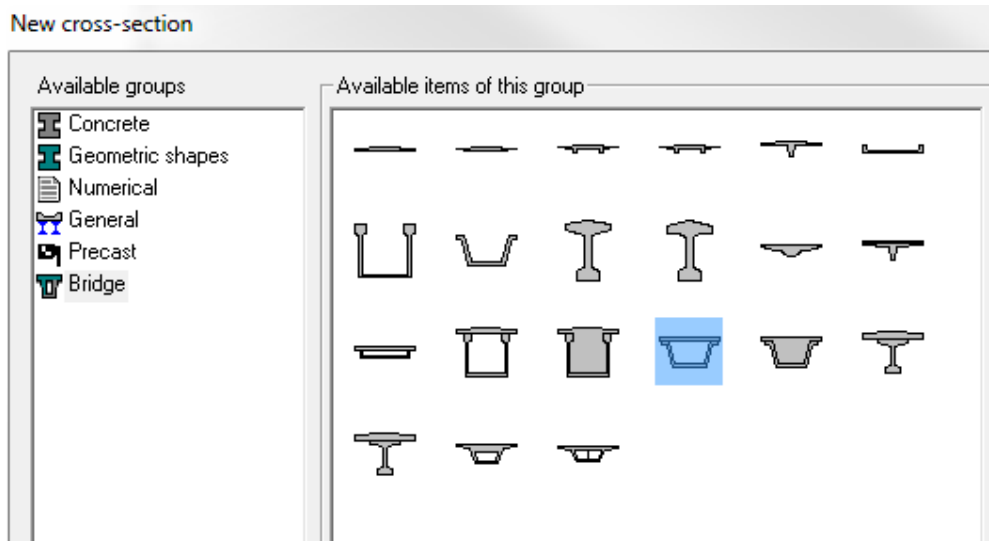
This goal of this chapter is to show how the time dependant analysis can be executed in SCIA Engineer. A calculation with time dependant analysis is necessary to determine the time dependant losses of the tendons. This can only be applied in a 2D environment.



It is possible and in most cases also acceptable to simplify a 3D plate model to a 2D beam model. It is most definitely acceptable if the length is much larger than the width of the plate. In such cases the analysis in longitudinal direction is much more important. The advantage of a 3D-model would be the forces that can be investigated in transverse direction. This will also be shown in the next chapters.

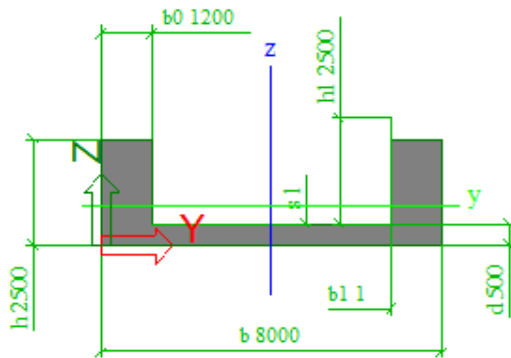
1_Input geometry and post-tensioning

The TDA analysis of post tensioned bridges is a much used application of the program. This is why there are many pre-parameterized bridges to be found in the cross-section library, which also makes the input of typical decks of bridges much easier.



Because this list is far from complete and does not contain all types of bridges, it is also possible to create a profile using 'General' cross-sections. The general cross-section opens a section-editor in where it is possible to manually create a cross-section, or to import one from dwg/dxf.

In this example we will use one of the predefined parametric bridges. The section used has the following measurements:



Parameters	
Material	C60/75
b [mm]	8000
b0 [mm]	1200
b1 [mm]	1
h [mm]	2500
h1 [mm]	2500
s [mm]	1
d [mm]	500
General	

We will insert the beam over a total length of 40m with intermediate supports at 10m and 30m.

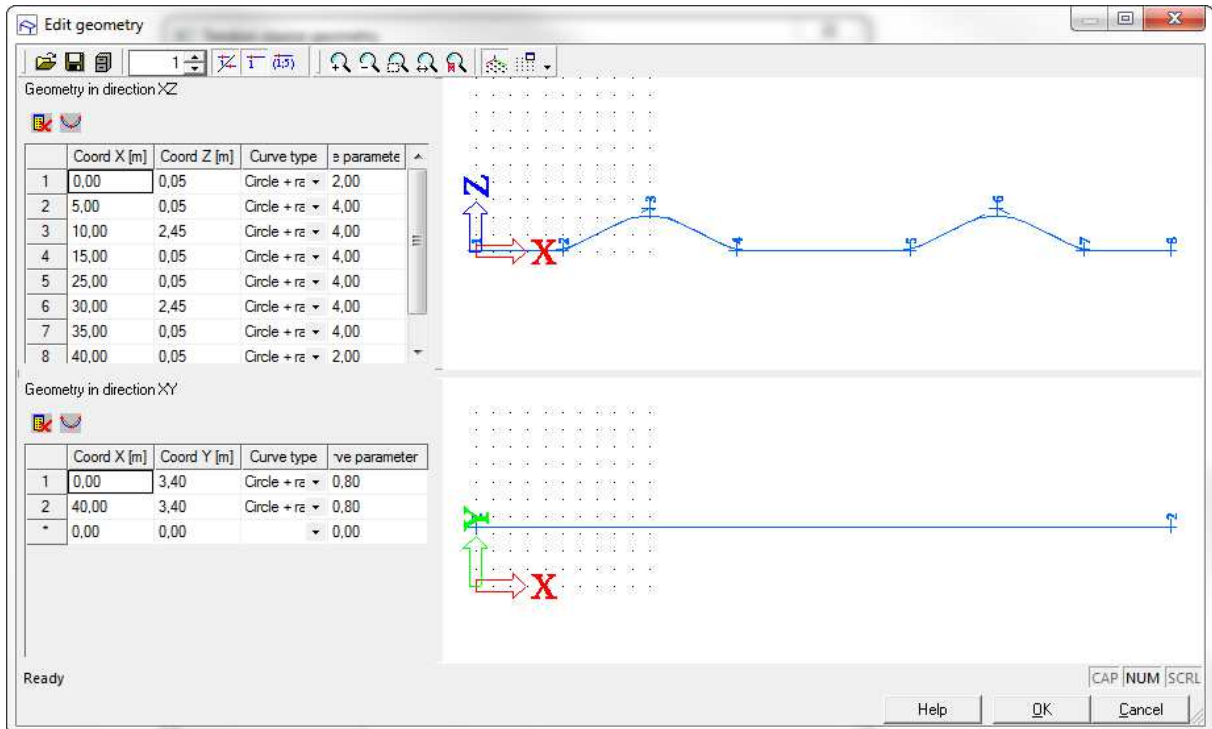
Adding the prestressed tendons is the most difficult part of the modeling. In the previous example this has been kept simple because it was possible to do this manually. In this example the input by source geometry will be used. This geometry can be inserted under :

'Libraries -> Prestressing -> Tendon source geometry'

In practice one usually starts from a dwg file which can be imported in SCIA Engineer and converted to a post-tensioned tendon. There is also more explanation about this as a demonstration movie on the site www.SCIA-online.com.

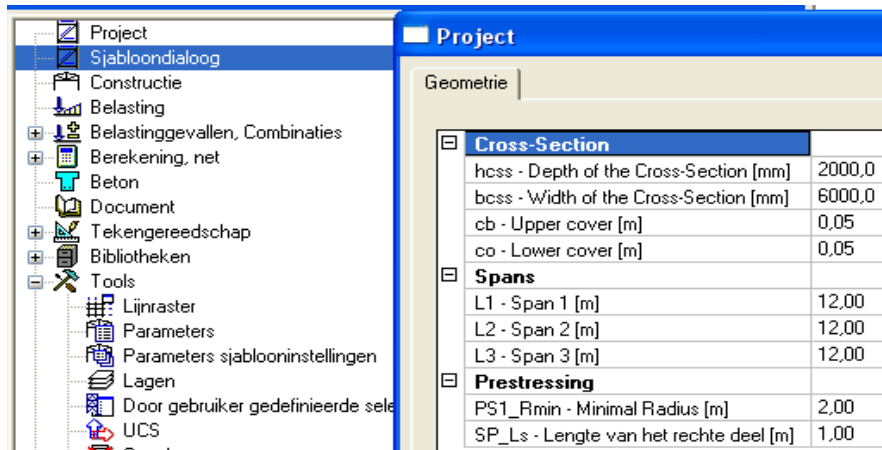
The source geometry allows us to determine the geometry of a post-tensioned tendon by using several points. The course of the tendon will be determined by means of interpolation between the points indicated by the user. For more explanation about the interpolation that takes place in the software, a reference is made to the documents for prestress under the help of SCIA Engineer.

The source geometry used in this example is shown in the image below.



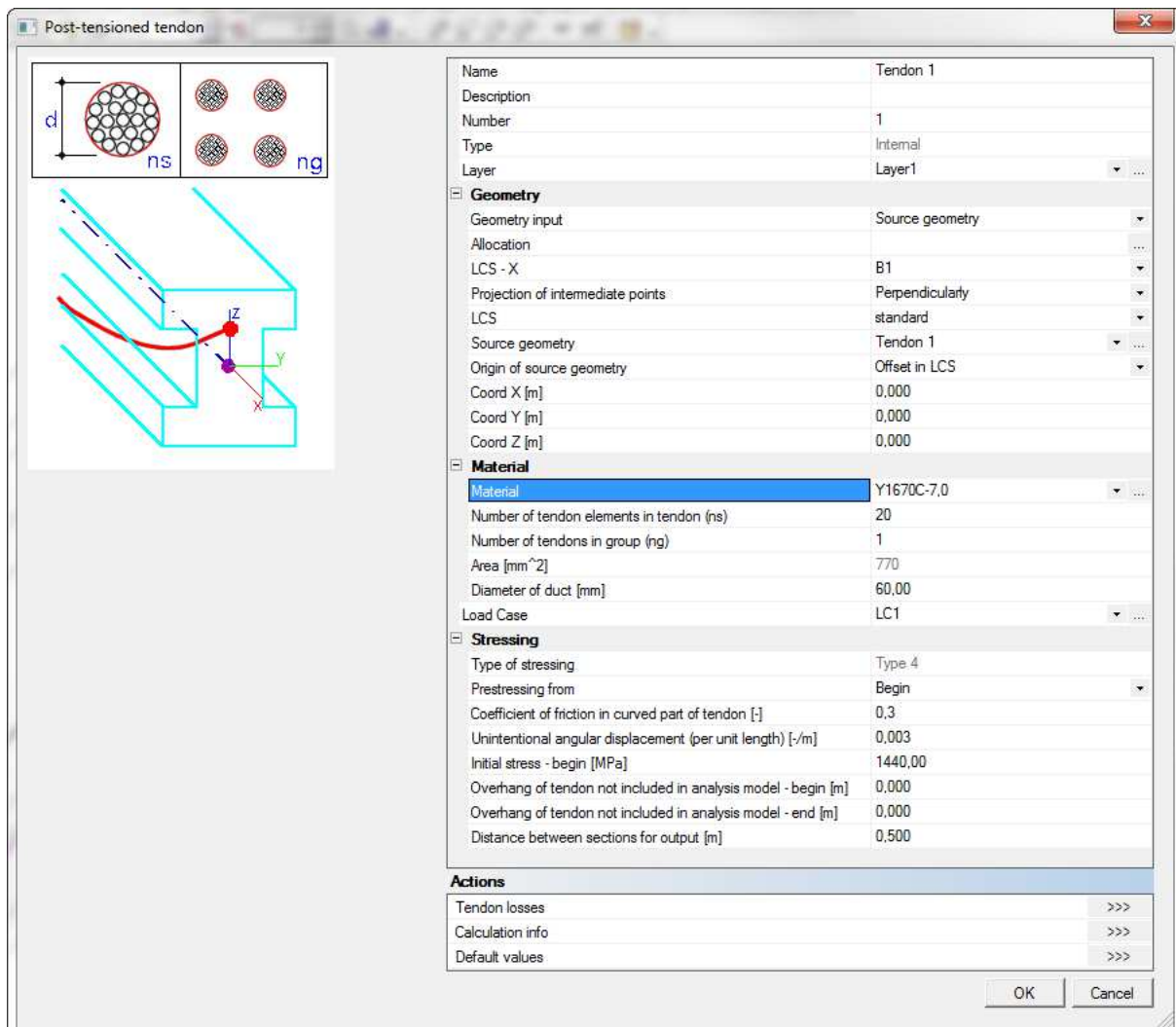
We will model two tendons in the bridge (one on the left side and one on the right side), and these two cables will be placed in the same load case 'LC1 – Prestress'. This means that the two cables will be tensioned at the same time.

Remark: For source geometry it can be very useful to use parameters. By using parameters the points of the source geometry can be easily determined with very limited input. An example of these input values can be seen in the image just below.



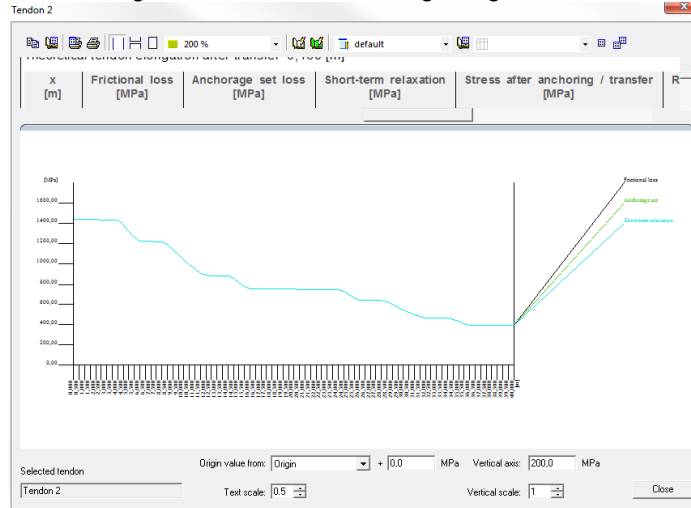
These input values are then linked to parameters that determine the coordinates of the source geometry.

The parameters of the tendon are determined as followed.



The system line of the member also has to be set at 'bottom'.

As the tendons are placed, it becomes possible to look at the immediate losses in the cable. This is in the assumption that the post-tensioning would be done at the beginning of the tendon.

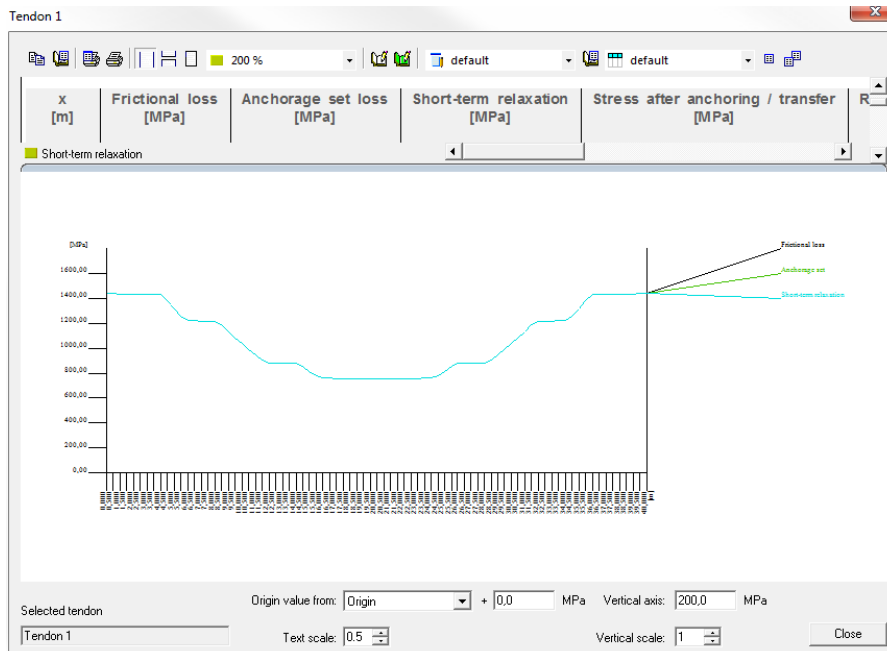


It is obvious that the loss due to friction has become too big at the end of the 40m long bridge. This is why it is advised that these cable are tensioned at both ends.

The following setting will be used for the tendons (in order to reduced the losses due to friction).

Stressing	
Type of stressing	Type 4
Prestressing from	Both ends, anchored at the end, restressed at the beginning
Coefficient of friction in curved part of tendon...	0,3
Unintentional angular displacement (per unit l...	0.003
Initial stress - begin [MPa]	1440,00
Initial stress - end [MPa]	1440,00
Overhang of tendon not included in analysis ...	0.000
Overhang of tendon not included in analysis ...	0.000
Distance between sections for output [m]	0.500

This results in the following losses:



2_Construction stages (and loads)

The construction stages of this project will not be normal linear construction stages, but will be part of a time dependant analysis. So for each construction stage, a certain time indication will be necessary.

Stage 1: Casting of the concrete bridge:	day 0
Stage 2: Adding the post-tensioned cable:	day 28
Stage 3: Adding the service load:	day 100
Stage 4: Check of the structure after 50 years	day 18000

The load cases taken into account will only be the self-weight and a service load. Also in accordance to the construction stages, which need a load case for each stage, we will need the following load cases.

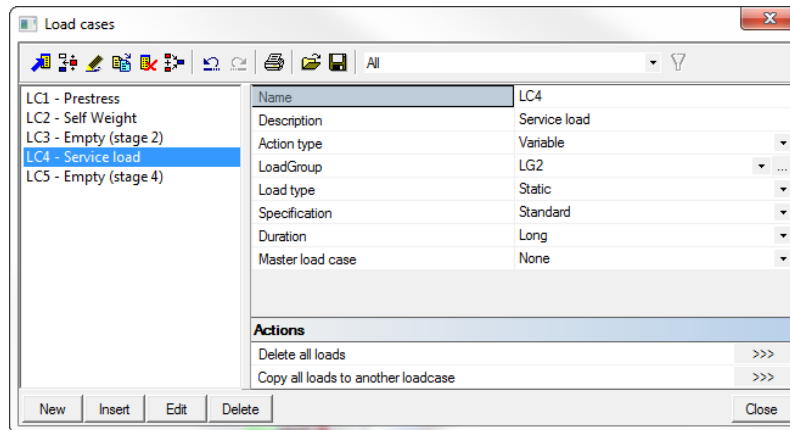
(LC 1: Prestress)

LC 2: Self weight

LC 3: Empty load case (for stage 2, where the post-tensioned cable will be added)

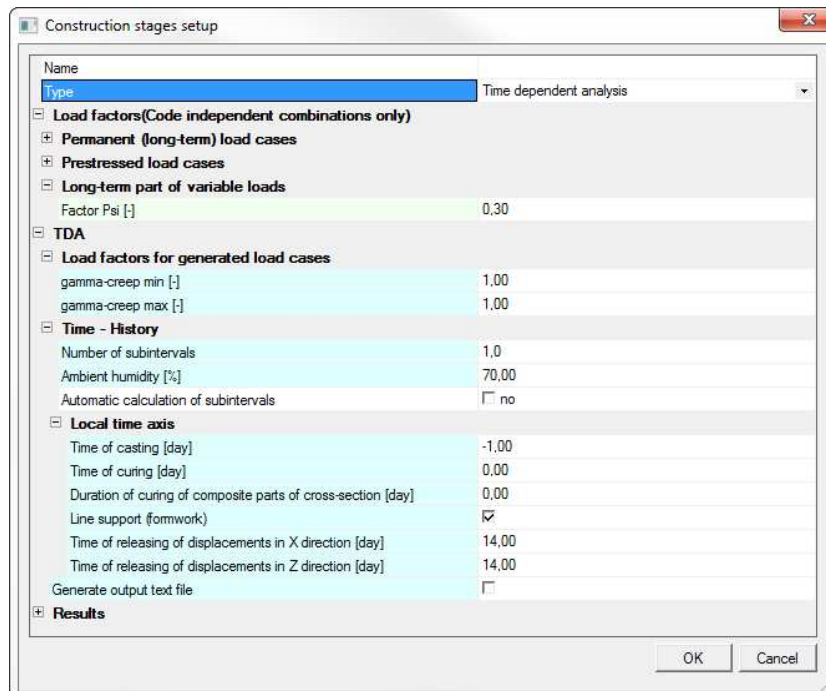
LC 4: Service load (variable load case, but of the type 'long term') → Line load of 10 kN/m

LC 5: Empty load case (for stage 5, the check after 50 years)



Remark: It is also possible to use mobile loads to create the load case. This is done to create the maximal moment in the middle of the span, or at the sides. For more information about mobile loads, a reference is made to the workshop about mobile loads.

The configuration for the time dependant analysis has to be as shown in the image below.



The most important properties are summed up.

Long-term part of the variable load: $\Psi = 0,3$

The variable loads are also taken into account for the time dependant analysis, but only for 30% (permanent loads are taken into account for 100%). This is only valid for variable loads of duration 'long'. The duration of a certain load case can be indicated in the configuration of the load case.

Number of subintervals (between 2 construction stages)

The numerical TDA method becomes more accurate if there are more subintervals between two construction stages. In return, more computational time is necessary. The number of subintervals is preferable given for each construction stage separately. For example, between the 3rd and 4th construction stage more subintervals are necessary because the time between them is much bigger.

Ambient humidity [%]

This property will affect the shrinkage. For higher ambient humidity, there will be less shrinkage.

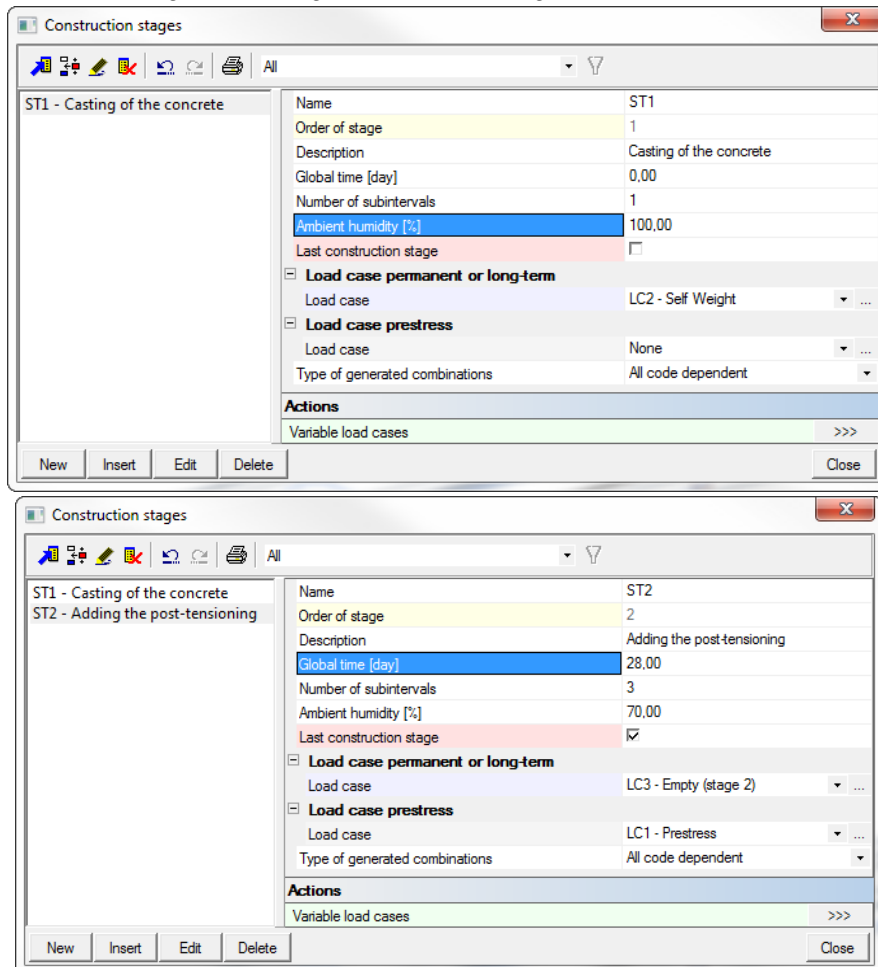
Local time axis

The local time axis introduces extra times for which no construction stage has to be defined:

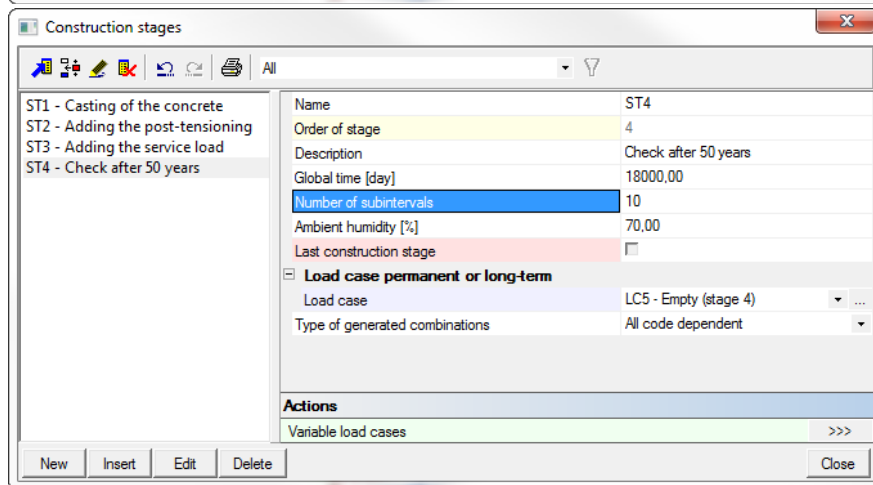
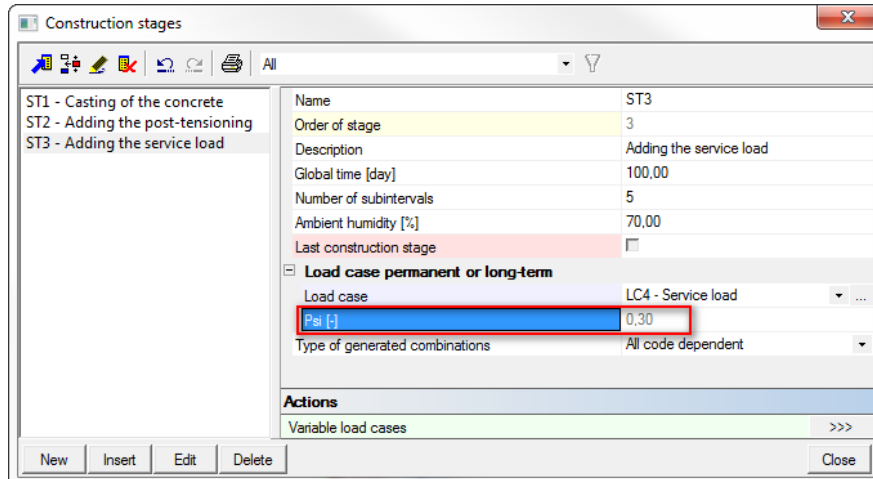
- *Time of casting:* This is preferably taken on day -1 to prevent the possibility that on day 0 an element without stiffness is sent to the solver (newly cast concrete has no stiffness).
- *Time of curing:* This is a special treatment of the concrete, which positively affects the shrinkage. It is mostly applied in the prefab industry.
- *Line support (formwork):* This property assumes that the concrete stays in the formwork after casting the concrete in order to build up stiffness before carrying any loads.

Remark: During this period the structure is completely supported and the deformations and internal forces do not occur.

The different construction stages are configured in the following manner.

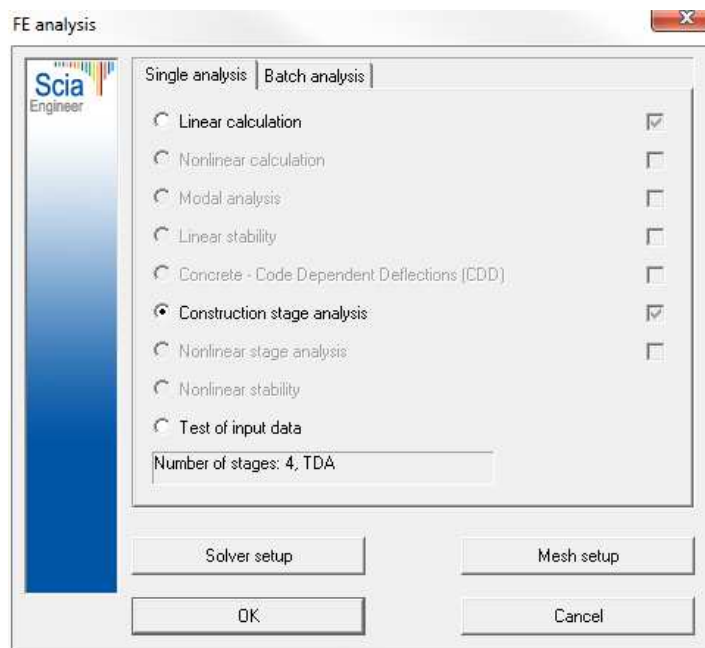


Note that after stage 2 the structure will not be changed anymore ('Last construction stage'). After this stage, long term variable load cases can be used as base for the construction stage.



3_Calculation

After the setup of the mesh and solver, the time-dependant analysis can be executed.



4_Results

The results which can be viewed are the in the same options as in the previous example.

Tendon stress

However, now it will also be possible to find long term losses. This effect is obvious in the tendon stresses for stage 3 and stage 4. The results are compared below.

Tendon stresses

Tendon stresses

Linear calculation, Extreme : Global

Selection : All

Tendons: All by selection

Class : ST3 (ULS)

Case	Tendon	x [m]	Stress after anchoring / transfer [MPa]	LED [MPa]	LCS [MPa]	Min Stress [MPa]	MaxStress [MPa]
ST3 (ULS)	Tendon 1	20,000	753,61	0,19	-13,37	740,43	740,43
ST3 (ULS)	Tendon 1	0,000	1440,00	0,00	-60,07	1379,93	1379,93

Tendon stresses

Tendon stresses

Linear calculation, Extreme : Global

Selection : All

Tendons: All by selection

Class : ST4 (ULS)

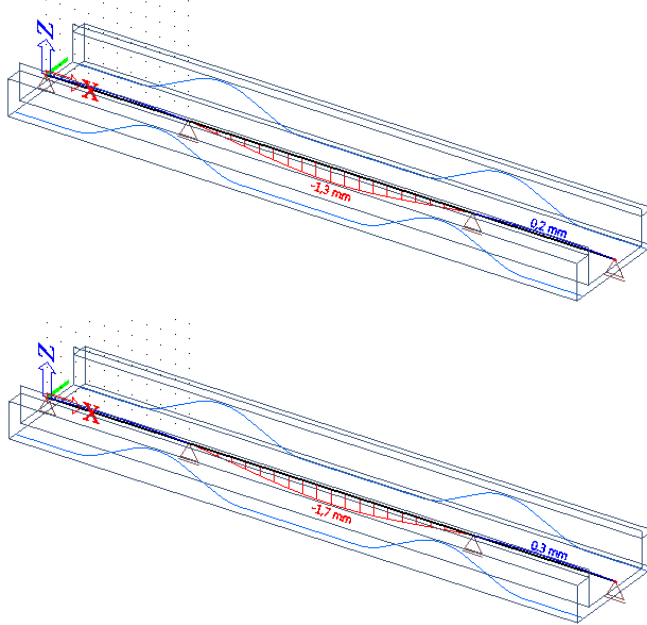
Case	Tendon	x [m]	Stress after anchoring / transfer [MPa]	LED [MPa]	LCS [MPa]	Min Stress [MPa]	MaxStress [MPa]
ST4 (ULS)	Tendon 1	20,000	753,61	0,19	-52,50	701,30	701,30
ST4 (ULS)	Tendon 1	0,000	1440,00	0,00	-159,20	1280,80	1280,80

It can be clearly seen that the long term losses, LCS (loss due to creep and shrinkage), are significantly higher after 50 years in comparison with the beginning of the last stage at 100 days.

The long term losses are also smaller at the middle part of the beam. This is because the tensioning after anchorage is lower in the middle part (due to anchorage losses), and the losses due to relaxation depend on the amount of stress in the tendon. So higher stress in the cable means higher long term losses.

Deformations

Aside from the tendons stresses, the deformations can also be inspected. Here we will also look at the difference between stage 3 and stage 4, to see the effects of the increased long term losses.



Properties

Deformations on member (1)

Name	Deformations on member
Selection	All
Type of loads	Combinations
Combinations	F3-EN-SLS Characteristic - Adding the service load
Filter	No
Structure	Initial
Values	uz
Extreme	Member
Drawing setup 1D	...
Section	All

Properties

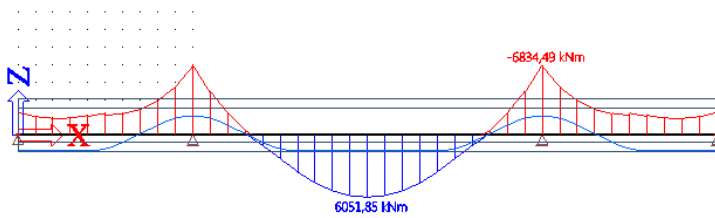
Deformations on member (1)

Name	Deformations on member
Selection	All
Type of loads	Combinations
Combinations	F4-EN-SLS Characteristic - Check after 50 years
Filter	No
Structure	Initial
Values	uz
Extreme	Member
Drawing setup 1D	...
Section	All

The deformation in the middle of the beam is 1,3mm at the beginning of the stage with the service load, and it is 1,7mm at the end of the same stage.

Moment diagram

The moment diagram after 50 years will look like:



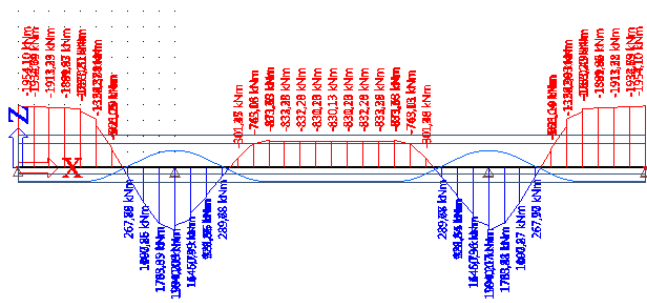
Properties

Internal forces on member (1)

Name	Internal forces on member
Selection	All
Type of loads	Combinations
Combinations	F4-EN-ULS (STR/GEO) Set B - Check after 50 years
Filter	No
Prestressing	<input type="checkbox"/>
Values	My
Extreme	Global
Drawing setup 1D	...
Section	All

With a moment of 6051.85kNm in the middle of the span and 6834,49kNm at the supports. The result comes from the combined loads of the self weight, service load and tensioning after 50 years.

The stress in the tendons creates an opposite moment (at day 100) of 830kNm in the middle of the span and 1994kNm at the supports (not including losses of stress in the tendon).



Properties

Internal forces on member (1)

Name	Internal forces on member
Selection	All
Type of loads	Load cases
Load cases	LC1 - Prestress
Filter	No
Structure	Initial
Prestressing	<input checked="" type="checkbox"/>
Type of forces	Total resultants
Values	My
Extreme	Section
Drawing setup 1D	...
Section	All

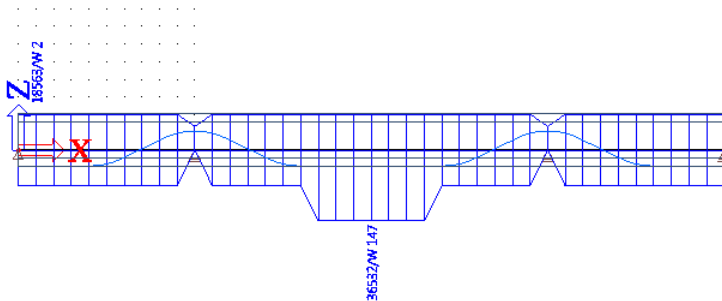
From these results we can expect that more post-tensioning will have to be added or more passive reinforcement will probably be required.

5 Checks in the concrete menu

Passive reinforcement

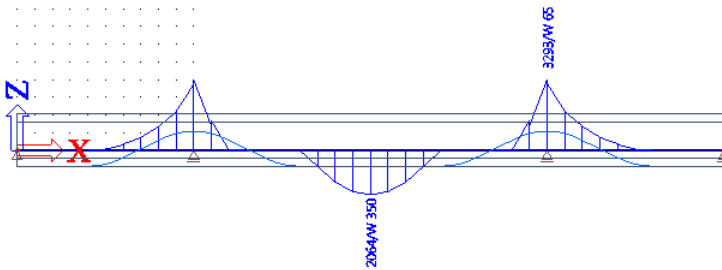
In the concrete menu the amount of passive reinforcement can be calculated.

The 'Concrete prestress design' of the prestressed beam will give the following result (while respecting the minimum reinforcement demand): 18 563 m³ of upper reinforcement and 36 532 mm³ of lower reinforcement.



Properties	
Concrete prestress design EN 1992-1-1 (1)	
Name	Concrete prestress design EN 1992-1-1
Selection	All
Type of loads	Combinations
Combinations	F4-EN-ULS (STR/GEO) Set B - Check after 50 years
Filter	No
Print explanation o...	<input type="checkbox"/>
Use named joints	<input type="checkbox"/>
Use named cuts	<input type="checkbox"/>
Values	As total req.
Extreme	Member
Drawing setup 1D	
Section	All

If the minimum reinforcement is not respected, then the following amount of reinforcement is calculated: 2 064 m³ of upper reinforcement and 3 293 mm³ of lower reinforcement.



Properties	
Concrete prestress design EN 1992-1-1 (1)	
Name	Concrete prestress design EN 1992-1-1
Selection	All
Type of loads	Combinations
Combinations	F4-EN-ULS (STR/GEO) Set B - Check after 50 years
Filter	No
Print explanation o...	<input type="checkbox"/>
Use named joints	<input type="checkbox"/>
Use named cuts	<input type="checkbox"/>
Values	As total req.
Extreme	Member
Drawing setup 1D	
Section	All

In the middle of the beam there will still be a certain amount of lower reinforcement necessary and above the supports there is still some upper reinforcement necessary. This means that the construction will become instable without any passive reinforcement.

Allowable stresses

The stresses in both the cables and the concrete must be checked to see if the prestress does not exceeds the maximal concrete pressure or the maximal tension in the tendons.

A. The check of allowable stress in the concrete after anchorage is easily suffices.

Allowable stress concrete EN 1992-1-1

Linear calculation, Extreme : Member

Selection : All

Combinations : F2-EN-ULS (STR/GEO) Set B

Prestress check of allowable stress concrete for selected members

Case	M _Z [kNm]	Member	d _x [m]	N [kN]	M _y [kNm]	σ _{c,aa} [MPa]	σ _{cc,min} [MPa]
					M _Z [kNm]	σ _{cc,max} [MPa]	σ _{cc,ch} [MPa]
F2-EN-ULS (STR/GEO) Set B/1	0,00	B1	14,000	-1476,97	556,01	-0,39	-0,39
					0,00	-36,00	0,00

B. The check of allowable tensile stress in the concrete of quasi-permanent loads also suffices.

Allowable stress concrete EN 1992-1-1

Linear calculation, Extreme : Member

Selection : All

Combinations : F4-EN-SLS Characteristic

Prestress check of allowable stress concrete for selected members

Case	M_z [kNm]	Member	d_x [m]	N [kN]	M_y [kNm]	$\sigma_{c,aa}$ [MPa]	$\sigma_{c,q,min}$ [MPa]	$\sigma_{c,q,max}$ [MPa]	$\sigma_{c,ch}$ [MPa]	$\sigma_{c,q,max}$ [MPa]	$\sigma_{c,lt,min}$ [MPa]
										$f_{ct,eff}$ [MPa]	$\sigma_{c,qp}$ [MPa]
F4-EN-SLS Characteristic/2	0,00	B1	10,000	-1523,83	-4450,57	-1,00	-1,01	0,00	0,00	1,78	-1,00
										5,16	0,00

The tensile stress occurs above the supports ($d_x = 10m$), but is not too big ($\sigma_{c,q,max} < f_{ct,eff}$).

C. The check of allowable tendon stress is also investigated.

Check of prestressing reinforcement

Linear calculation, Extreme : Global

Selection : All

Tendons: All by selection

Combinations : F2-EN-ULS (STR/GEO) Set B

Check of prestressing reinforcement for selected tendons

Tendon	d_x [m]	Case	$\sigma_{p,pa}$ [MPa]	$\sigma_{p,aa}$ [MPa]	$\sigma_{p,q,min}$ [MPa]	$\sigma_{p,q,max}$ [MPa]	$\sigma_{p,lt}$ [MPa]	Check _{cal} [-]
			$\sigma_{p,max}$ [MPa]	$\sigma_{p,m0}$ [MPa]	$\sigma_{p,m}$ [MPa]	$\sigma_{p,m}$ [MPa]	$\sigma_{p,m}$ [MPa]	Check _{lim} [-]
Tendon 1	0,00	F2-EN-ULS (STR/GEO) Set B/1	1440,00	1440,00	1440,00	1440,00	1440,00	1,18
		ST2	1296,00	1224,00	1252,50	1252,50	1252,50	1,00

The tendon stress before and after anchorage exceeds the allowable stresses. In such a case we will want to keep the total compressive stress in the concrete, but we want to stay under the maximum allowed.

A possible solution is to use a material which has a higher maximal tendon stress, or to lower the tendon stress in the tendons themselves. This can be done by using more tendons than in the current situation.

The tendon stress after 50 years will be lower, so it is advisable to keep the same total amount of prestress, because if the total amount of prestress would be lowered, it is possible that the previous checks (especially the allowable tensile stress in the concrete) will not pass anymore.

Linear analysis of a post-tensioned bridge deck

In this chapter post-tensioned cables are introduced on a bridge deck. This deck will be modeled as a 2D-element, allowing the calculation to run in a general XYZ environment. As previously mentioned a time dependent analysis cannot be performed in the general XYZ environment. In order to take into account a certain long-term loss, the user itself would be able to make an estimate of this loss and simply include this by giving the prestressed cables a lower initial stress.

It might be possible to tension the cables sequentially and to calculate these losses by using linear construction stages. The process is analogous to example 4.

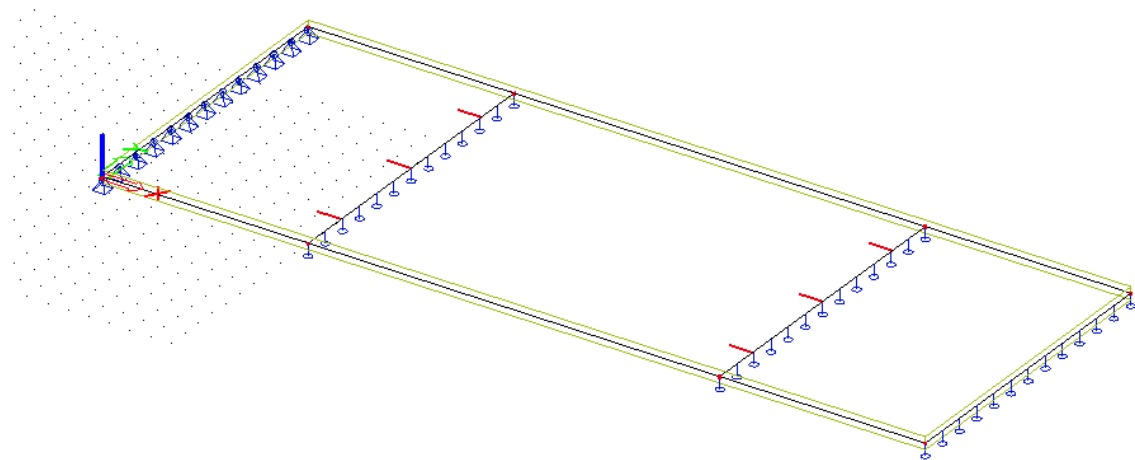
This example will be restricted to a linear analysis of a post-tensioned bridge deck without working with construction phases.

1_Input geometry and post-tensioning

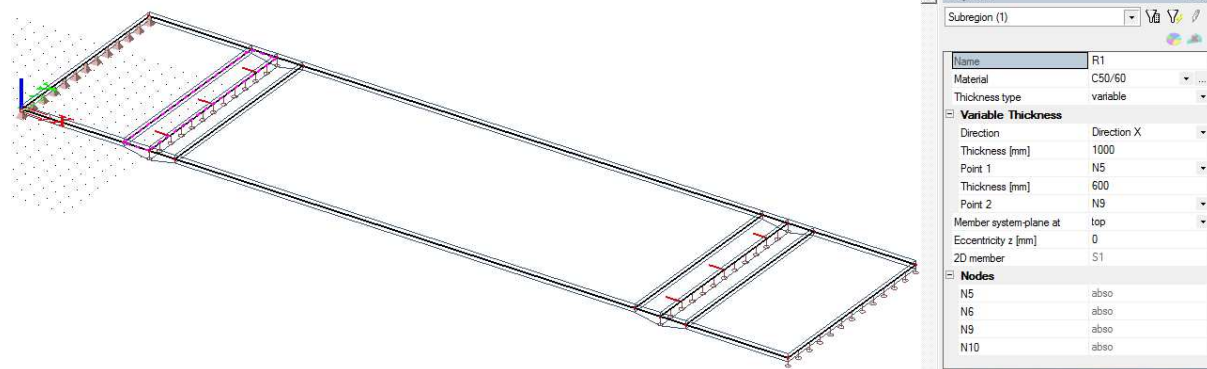
Before the modeling can start, the project data must be inputted. We will choose for a general XYZ environment with material concrete C50/60. In the functionalities the option 'Prestressing' is ticked on (and on the right side, 'Advanced parameters', to indicate for example anchorage losses).

The plate has a size of 40 by 15 meters and a thickness of 600mm.

It is supported by supports at the ends and at 10m distance from the ends. To introduce a line support at 10m distance from the ends, you will first have to introduce an internal edge. These steps result in the following structure:



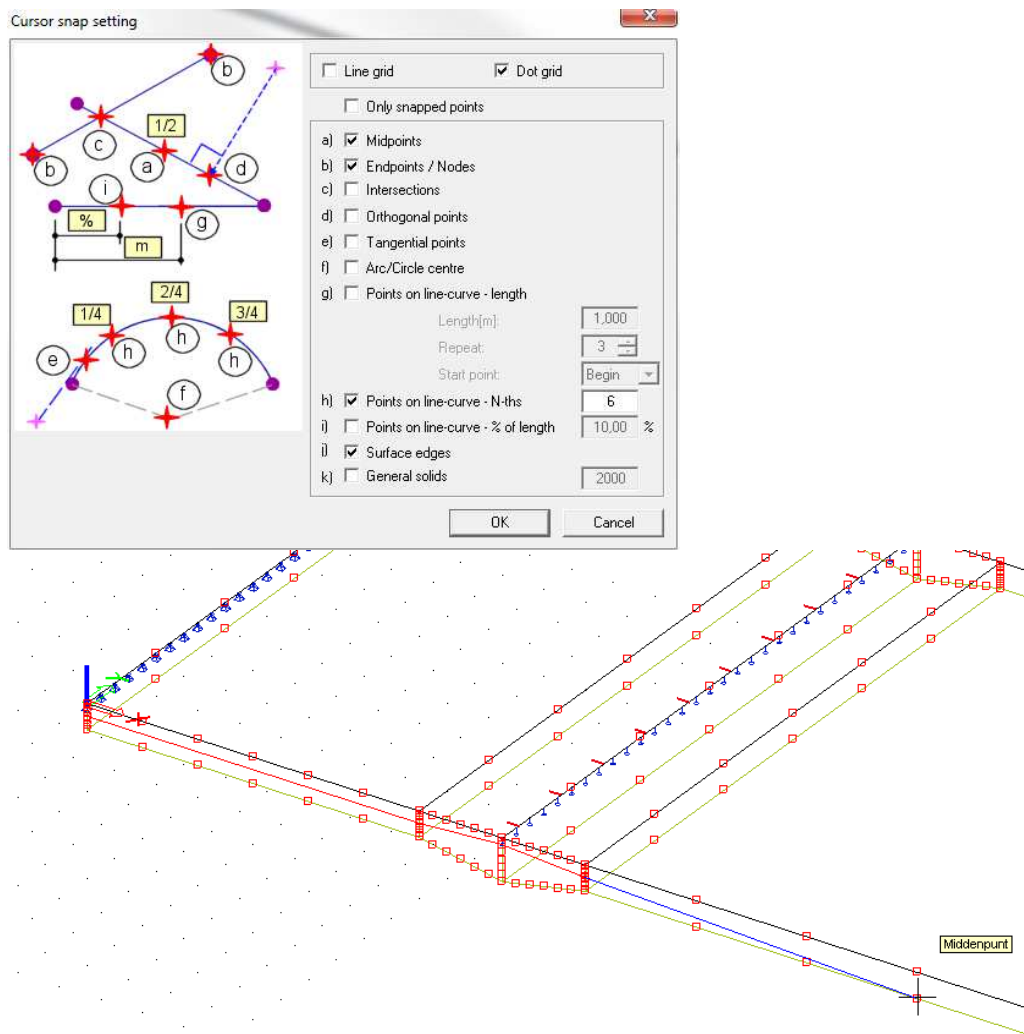
The next step is to create subregions above the internal supports in order to locally thicken the plate. At a distance of 2m from the intermediate support, the thickness is increased from 600mm to 1000mm (towards the support in question). Also the alignment is set to 'top' (so that the top surfaces are at the same height).



Applying the post-tensioned cables is completely analogous as in 1D elements. Again, there is a possibility of direct input, input via source geometry or starting from a dwg-file. The direct input or import via source geometry could be simplified by means of parameters.

Material	
Material	Y1770S7-9
Number of tendon elements in tendon (ns)	20
Number of tendons in group (ng)	1
Area [mm ²]	1000
Diameter of duct [mm]	60,00
Load Case	
Load Case	LC1 - Prestress
Stressing	
Type of stressing	Type 4
Prestressing from	Both ends, anchored at the end, restressed at the beginning
Coefficient of friction in curved part of tendon [-]	0,3
Unintentional angular displacement (per unit length) [-/m]	0,003
Anchorage set - begin [mm]	6,00
Anchorage set - end [mm]	6,00
Initial stress - begin [MPa]	1440,00
Initial stress - end [MPa]	1440,00
Overhang of tendon not included in analysis model - begin [m]	0,000
Overhang of tendon not included in analysis model - end [m]	0,000
Distance between sections for output [m]	0,500

In this example, 1 cable is inserted through direct input. Then this cable is copied several times over the width of the plate. The direct input is done using snapped points. The coordinates of the snapped points may subsequently be adjusted according to 'table edit geometry'.



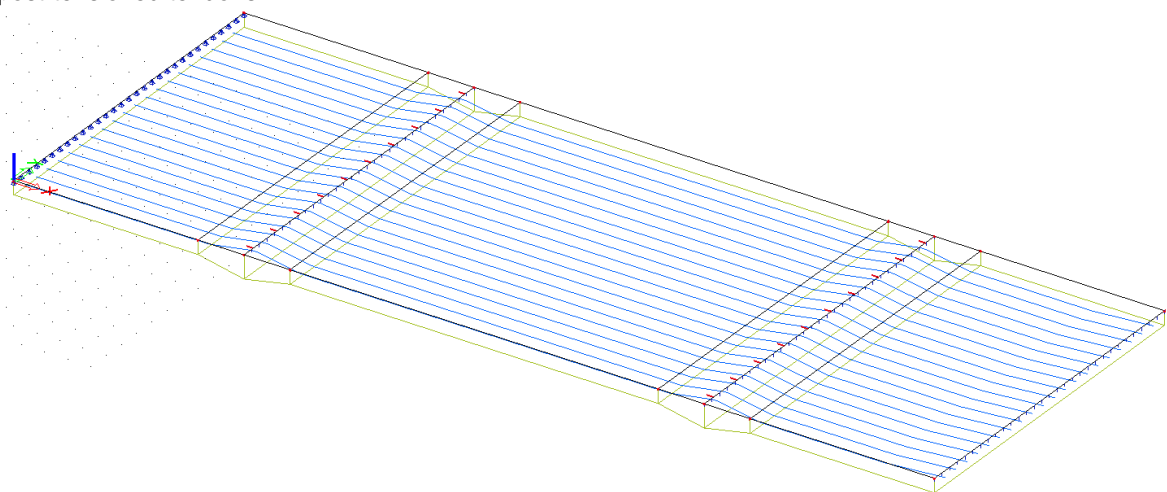
The coordinates of the snapped points may subsequently be adjusted according to 'table edit geometry'. This way the course of the tendon can be numerically perfected.

	Coord X [m]	Coord Y [m]	Coord Z [m]	Curve type	Curve parameter [m]
1	0,000	0,000	-0,300		
2	8,000	0,000	-0,300	Circle + radius	1,000
3	10,000	0,000	-0,167	Circle + radius	1,000
4	12,000	0,000	-0,300	Circle + radius	1,000
5	20,000	0,000	-0,550	Circle + radius	1,000
6	28,000	0,000	-0,300	Circle + radius	1,000
7	30,000	0,000	-0,167	Circle + radius	1,000
8	32,000	0,000	-0,300	Circle + radius	1,000
9	40,000	0,000	-0,300		

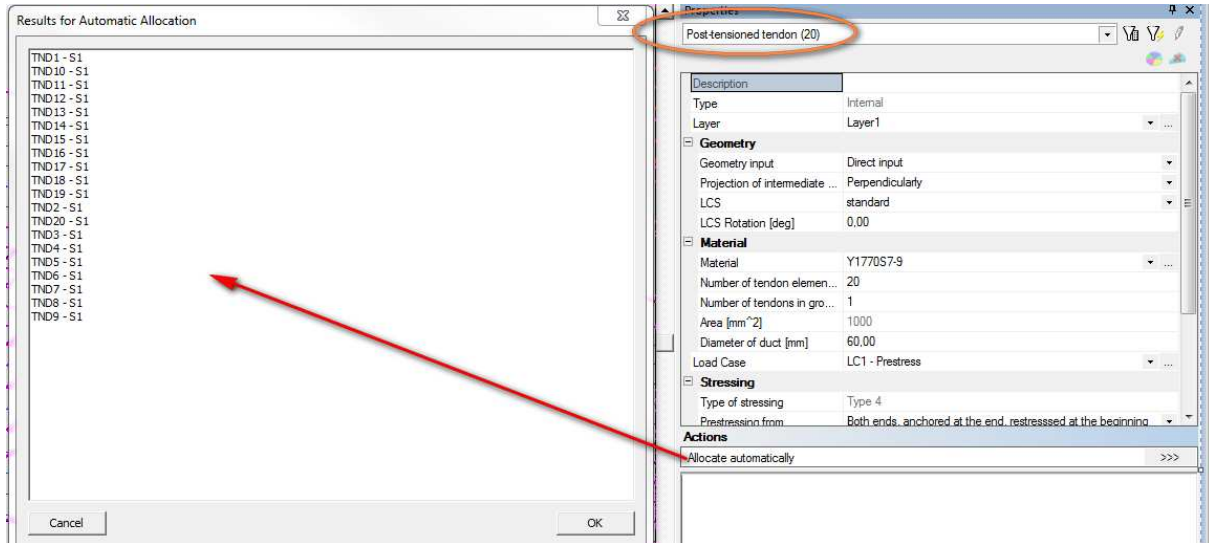
If the first tendon is placed according to wish, it can be copied 20 times over the width of the plate.

Number of copies	21	Connect selected nodes with new beams	<input type="checkbox"/>
<input checked="" type="checkbox"/> Insert the very last copy		Copy additional data	<input checked="" type="checkbox"/>
Distance vector		How to define the distance ?	
Define distance by cursor	<input type="checkbox"/>	<input type="radio"/> between two copies <input checked="" type="radio"/> from original to the last copy	
x	0,000 m	How to define the rotation ?	
y	15 m	<input checked="" type="radio"/> between two copies <input type="radio"/> from original to the last copy	
z	0,000 m	Rotation around	
rx	0,00 deg	<input checked="" type="radio"/> current UCS <input type="radio"/> distance vector	
ry	0,00 deg		
rz	0,00 deg		
		OK	Cancel

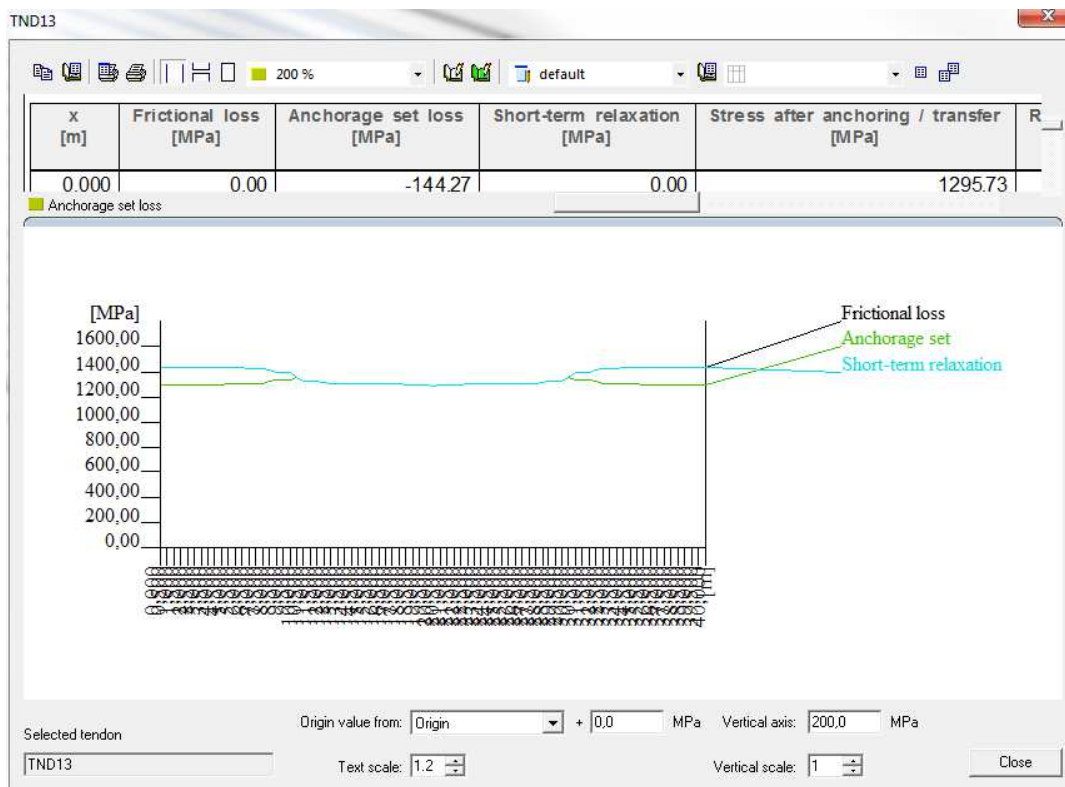
After copying the tendon, there are now 22 tendons in the plate, including the tendons at Y=0 and at Y=20. These outer tendon are not present in reality, so they are deleted. The result is a plate with 20 post-tensioned tendons.



After entering the post-tensioned cables, they still need to be assigned to the plate. This can be done for all cables simultaneously through the action 'automatically assign'.



These are the tendon stress losses in one of the tendons:



2 Input of the loads

In this example two additional load cases are created:

(LC 1: Prestress)

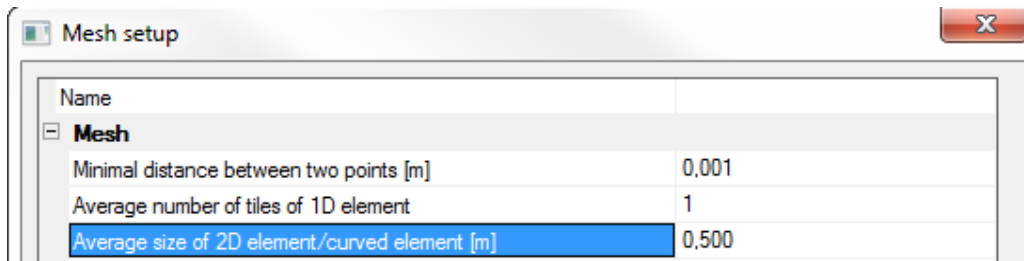
LC 2: Self Weight

LC 3: Service Load → a surface load of 5 kN/m² applied over the entire plate

After the load cases and loads have been introduced, an ULS and a SLS combination are made.

3_Calculation

Before the calculation can be performed, the size of the mesh must first be set. The calculation will be done with a mesh size of 0,5 m and maintain also this mesh size for the prestressed cables.



After changing this setting, the linear calculation can be performed.

4_Results

The results are available after executing the linear calculation.

Since no construction stages are being used, the tendon stresses will remain the same as the stress after anchoring, which already could be determined from the linear calculation.

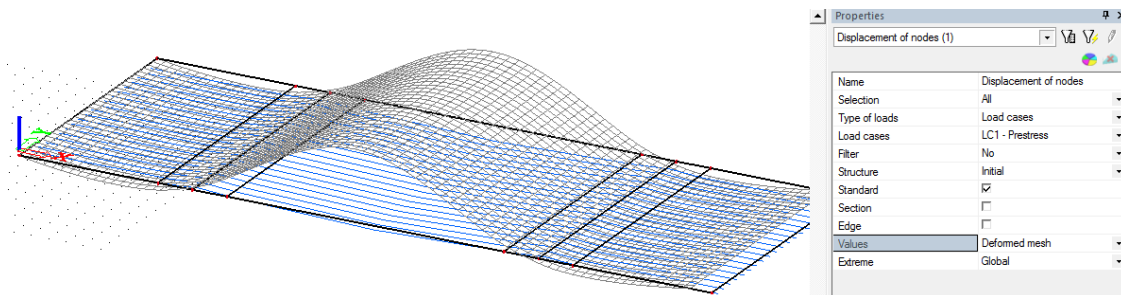
After evaluation of the results, the user can see by means of deformation and internal forces whether or not the optimal post-tensioning has been applied.

This is a general comment on the use of pre-tensioning in SCIA Engineer. The program allows a very complex analysis to be carried out, but it does not design the amount of pre-tensioning itself. The program only does the analysis. It is the responsibility of the user to evaluate the results, in other words whether the applied pre-tensioning may or may not be optimal.

Viewing the deformed structure under prestressed circumstances gives the user an idea if the course of the cable is chosen optimally.

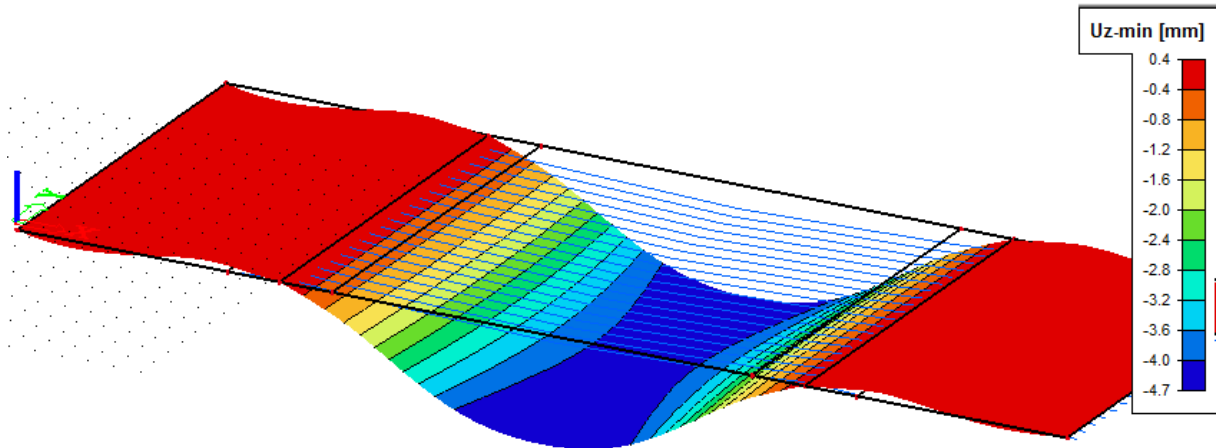
In this construction, the deflection in the middle of the field will be the most disadvantageous. Post-tensioning must be designed in such a way that the deflection in the middle of the field is prevented.

The deformation under post-tensioning is shown below:



This deformation is clearly the opposite of the deformation under its self weight.

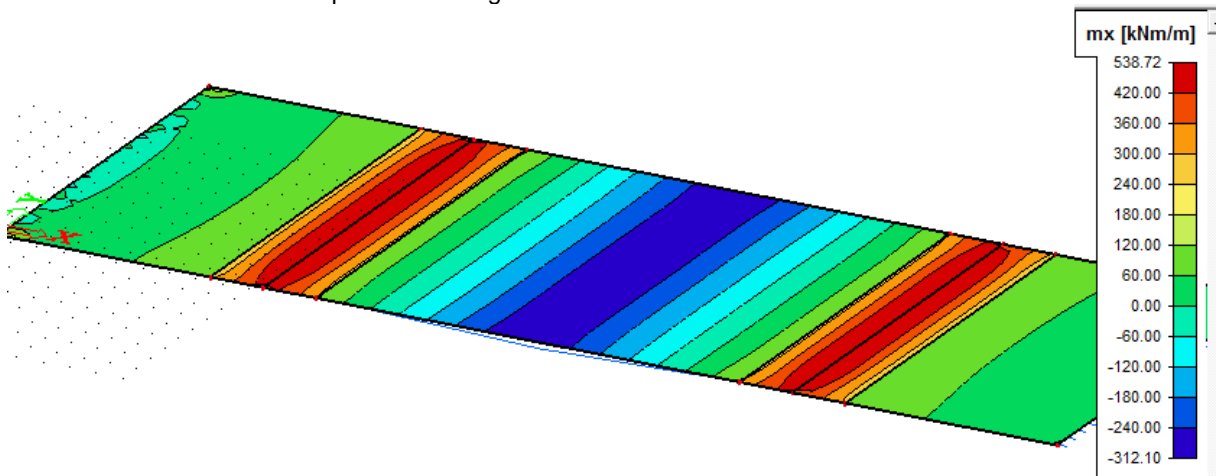
The deformation of the (characteristic) SLS combination shows the resulting deformation of the post-tensioned structure in charge of its self weight and service load.



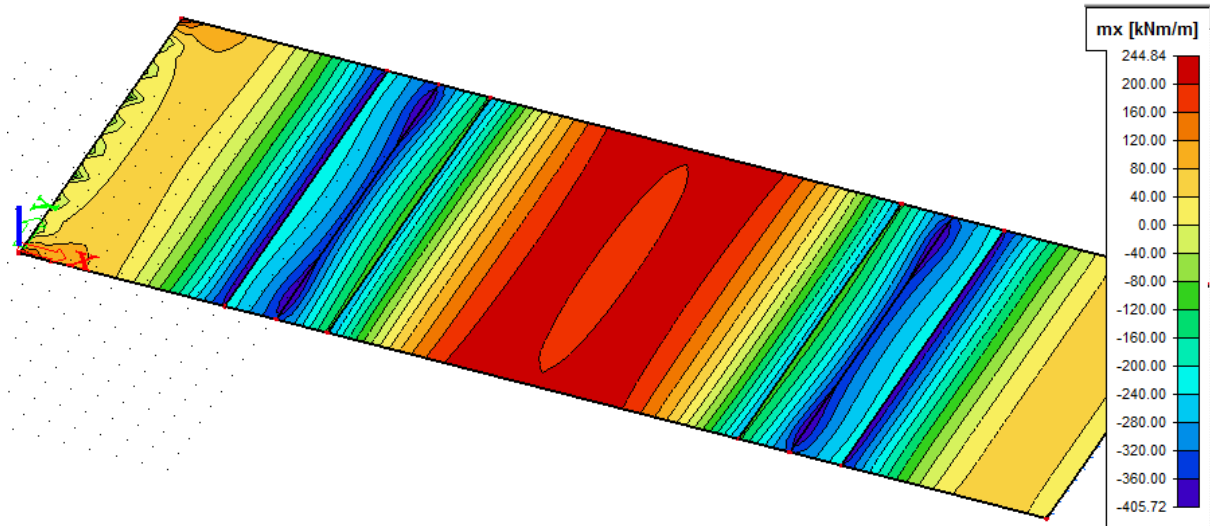
The course of the cable is well chosen because it counteracts the deformation of the self weight and the service load.

The same applies to the internal forces. The moment m_x , as a result of post-tensioning, is clearly opposite to that of its self weight and service load.

The moment m_x due to post-tensioning is shown below.



The moment m_x for the ULS combination looks as follows:



Remark: Because the ULS is an enveloping combination and therefore only an enveloping result would be retrieved, only the following linear ULS combination is shown here:

$$1 \cdot \text{LC1} + 1,35 \cdot \text{LC2} + 1,5 \cdot \text{LC3}$$

The user should continue to evaluate if the resulting internal forces can be absorbed by the concrete itself or by passive reinforcement. If he considers that this is not the case, then the prestressing force can be increased and the linear calculation can be performed again.

The design of the passive reinforcement and also the controls for allowable stresses are supported only for 1D elements.

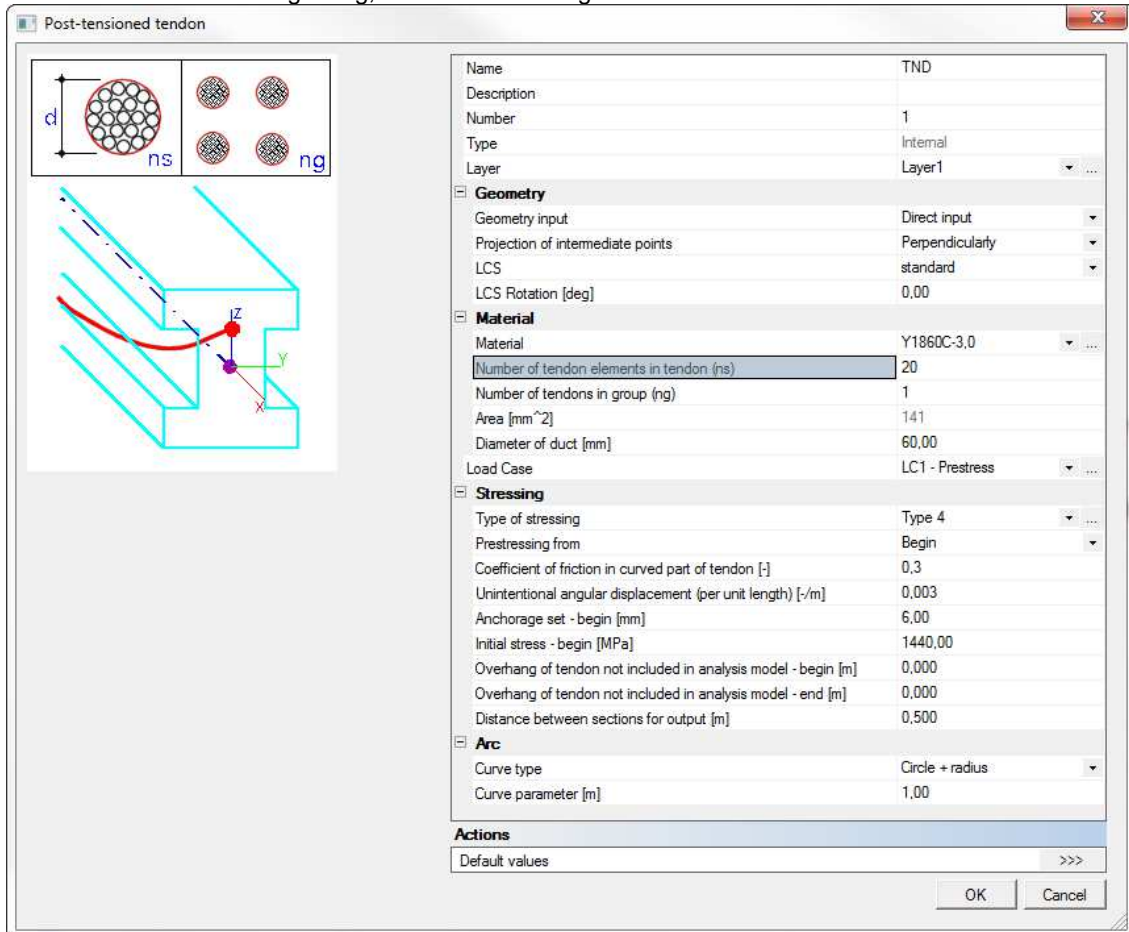
Analysis of a slab post-tensioned in 2 directions

Post-tensioned cables can also be applied on 2D elements. This means that the post-tensioning can be applied in two directions. Below, an example of a post-tensioned slab in 2 directions will be discussed.

1_Input of geometry and post-tension

A square plate with a thickness of 400mm is chosen (with 'Member system-plane at = centre'). The span is 10 meters and at each edge a hinged line support is added (so the load is carried to the supports in 2 directions). The concrete quality C30/37 is chosen.

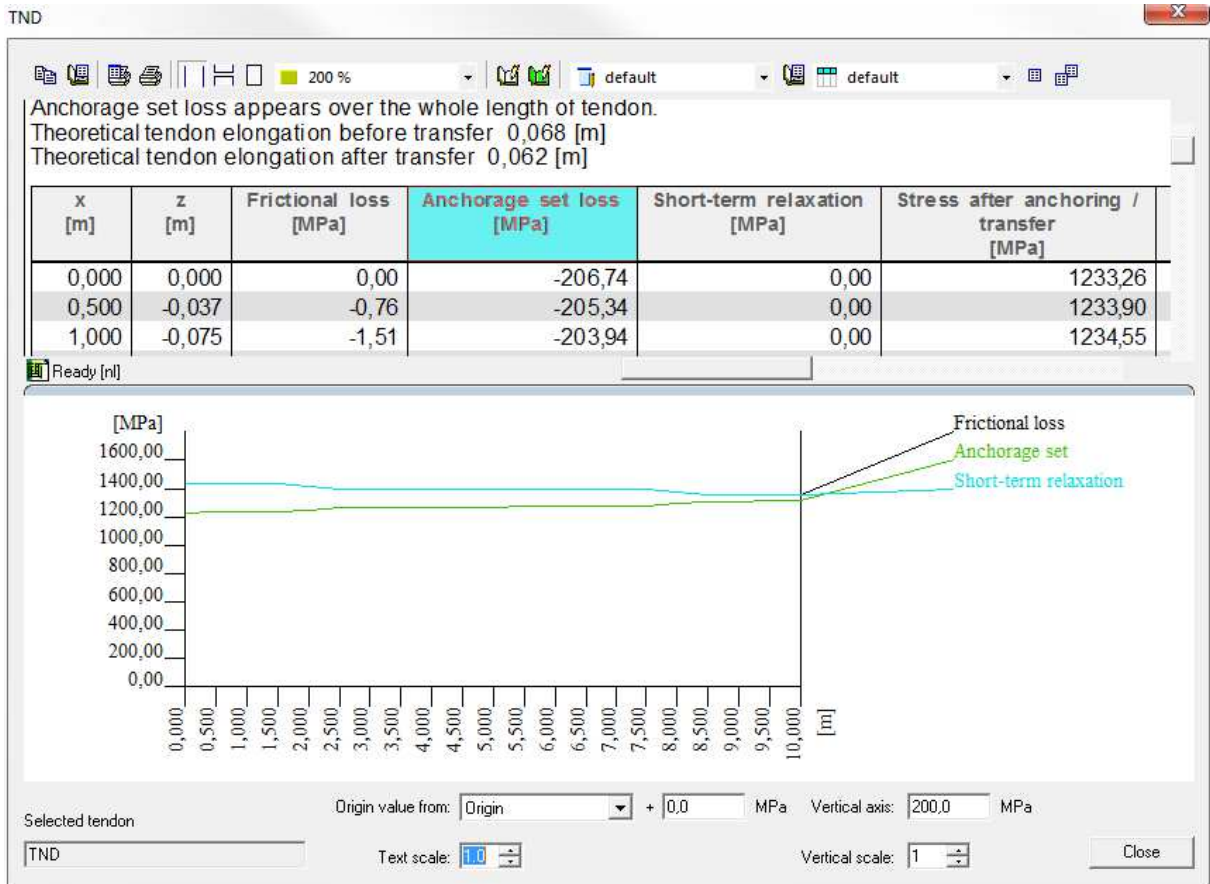
Next the tendons have to be inputted. The properties are those as seen in the image below. The cables are tensioned from the beginning, without restressing at the end.



The cables will be placed with an interval of 1 meter in each direction. And the geometry of the cables follows the coordinates as follows (the tendons are heightened towards the edges of the plate):

	Coord X [m]	Coord Y [m]	Coord Z [m]	Curve type	ve parameter
1	0,500	0,000	0,000		
2	0,500	2,000	-0,150	Circle + radius	1,000
3	0,500	8,000	-0,150	Circle + radius	1,000
4	0,500	10,000	0		

This gives the following result for the course of the cable losses:

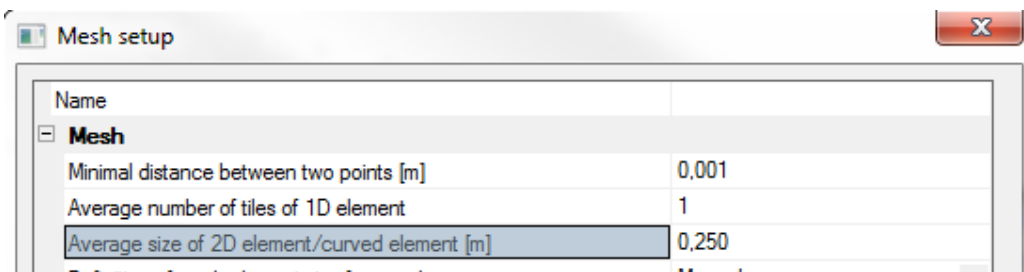


2_Input of loads

Since this is a very brief example, only the load case of 'Prestress' is shown. The effect of the prestress of each tendon is combined in load case LC1 for direction X and Y. It would also be possible to divide this in separate load cases, to examine the contribution of the pre-tensioning individually for each direction. And by using construction stages, it is also possible to simulate the tensioning sequentially.

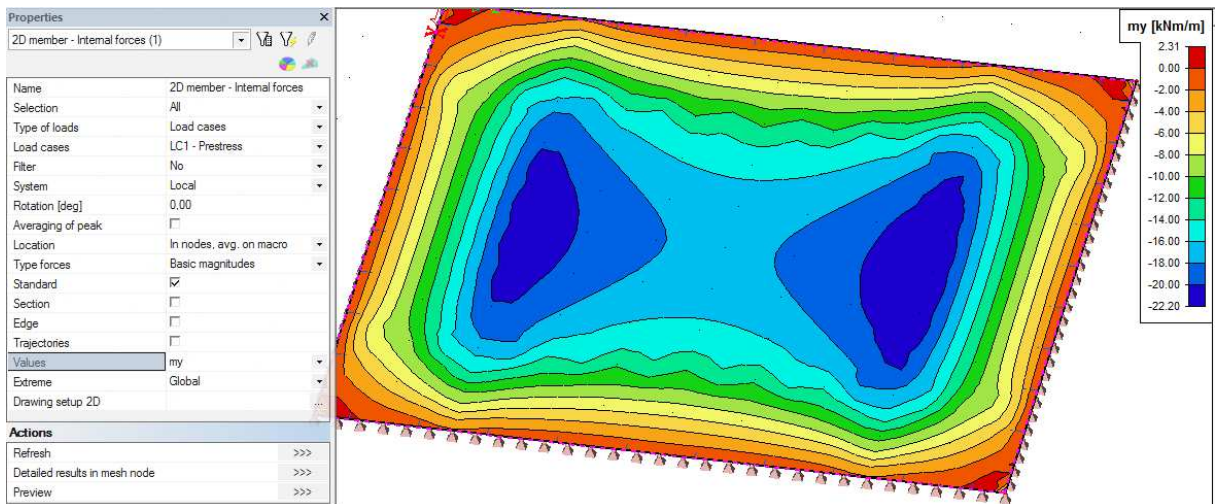
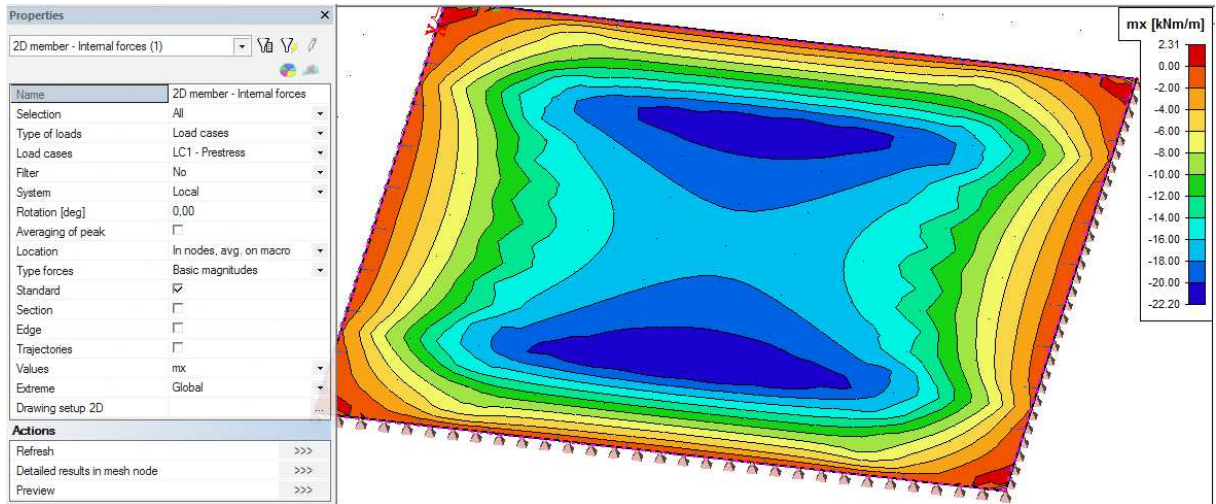
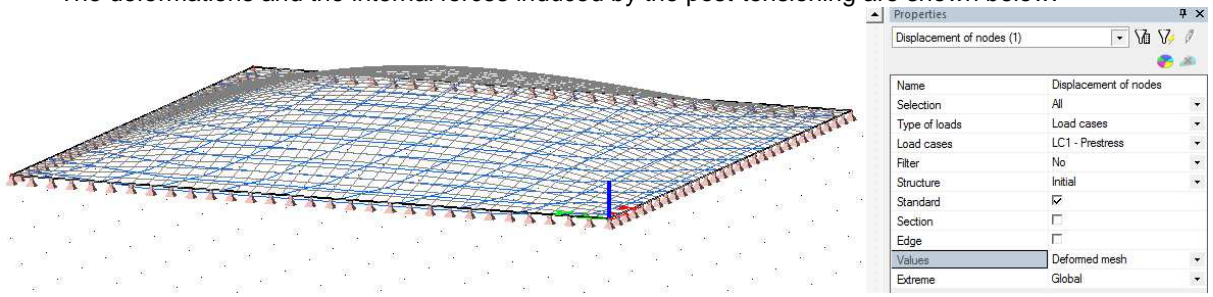
3_Calculation

The mesh size for 2D elements is set at 0,25m. Then a linear calculation is executed.



4_Results

The deformations and the internal forces induced by the post-tensioning are shown below.



For the moments, it is clear that symmetry is present in the prestress effects. This points out that the tendons are taken into account for both directions.

For an interpretation of the results, reference is made to the preceding examples.

Detailed documents about design and checks of post-tensioned slabs

For a more detailed document about post-tensioned plates, a reference is made to the tutorial 'Post-tensioned concrete slab EN1992-1-1'.

This tutorial elaborates on the design and checks of post-tensioned slab elements according to EN 1992-1-1. The focus is not so much on imports as in previous chapters, but on the discussion of the output.

This tutorial is only available in English.

Theoretical background – types of prestressing

This text comes from the book
“Prestressed Concrete Structures”, written by Jaroslav Navrátil, 2006, published by
AKADEMICKÉ NAKLADATELSTVÍ CERM, s.r.o.

Loss of prestressing due to relaxation of prestressing reinforcement

The loss of prestressing due to relaxation of the prestressing reinforcement applies to both production and service stages of the prestressed structure. The nature of this phenomenon was described in chapter 2.3.2. It was stated there that the size of relaxation depends on the (i) level of introduced prestressing and (ii) time. The calculation of relaxation must therefore take into account the procedure of stressing, anchoring and losses of prestressing due to other effects. Considering the fact that the relaxation of prestressing reinforcement is a negative phenomenon (it reduces the prestressing), we try to minimise its size by (i) the application of special materials (low-relaxation strands) and (ii) a special production process termed *correction of relaxation through keeping the stress constant* during stressing, see Fig. 4-13.

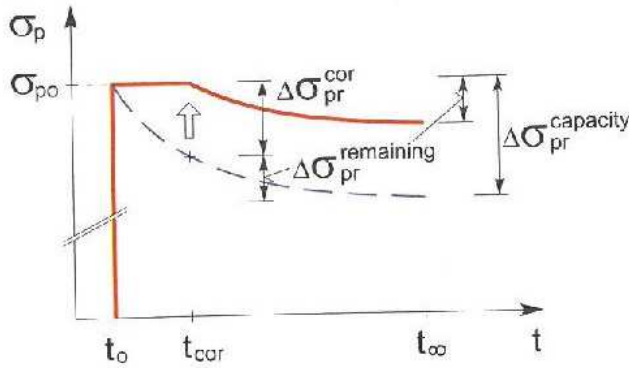


Fig. 4-13 Correction of relaxation through keeping constant stress during stressing

Stress σ_{p0} is kept during the stressing for a short time ($t_{cor} - t_0 = 2$ to 10 minutes) on a constant level with the creep of reinforcement taking place during this period, see Fig. 2-17. As the increase of relaxation is dramatic at the beginning of loading, even this short time period leads to a relatively significant reduction of relaxation capacity $\Delta\sigma_{pr}^{capacity}$ (the total loss of prestressing due to relaxation at time infinity). Relaxation $\Delta\sigma_{pr}^{cor}$ equivalent to the creep that occurs during the time when the stress is kept

constant can be calculated, similarly to the loss of prestressing due to relaxation, using what is known as relaxation tables. As an example, let us calculate the relaxation of stress relieved (Class 2) strands with characteristic tensile strength $f_{pk} = 1\,800$ MPa, and characteristic 0.2% proof-stress $f_{0.2} = 1530$ MPa), in accordance with EN 1992-1-1, 2004.

EN 1992-1-1 standard does not define the relaxation tables (see chapter 2.3.2) explicitly. Both the total relaxation at infinite time and the development of the relaxation loss over time can be calculated directly from formulas (3.28) to (3.30) of the standard. Specifically for Class 2 the ratio of the relaxation loss over the stress in prestressing steel is calculated as

$$\frac{\Delta\sigma_{pr}}{\sigma_p} = 0.66 \rho_{1000} e^{3.1\mu} \left(\frac{t}{1000}\right)^{0.75(1-\mu)} \cdot 10^{-5} \quad (4.37)$$

in which t is time in hours, μ is the ratio σ_p / f_{pk} , and ρ_{1000} is the value of relaxation loss (in %) at 1 000 hours after tensioning at a mean temperature of 20°C. The formula can be used for the determination of relaxation tables. The total relaxation with respect to a relative stress can be calculated by substituting “infinite” time 500 000 hours. For $\mu = 2.5\%$ (Class 2) we obtain the values in column (2) of Table 4-4 (a). For comparison, the related values according to standard ČSN 73 6207, 1993, Table 30 are shown in column (2) of Table 4-4 (b).

Table 4-4 Example of (part of) relaxation tables

(a) EN 1992-1-1, 2004

σ_p/f_{pk}	Total relaxation	t [min]	Dvp. of relax. for $\mu=0.7$	Dvp. of relax. for $\mu=0.8$
(1)	(2)	(3)	(4)	(5)
0.6	0.025	2	0.0243	0.0839
0.7	0.039	3	0.0266	0.0891
0.8	0.061	5	0.0298	0.0962
0.9	0.095	10	0.0349	0.1068
1.0	0.148	1 year	0.4025	0.5452

(b) ČSN 73 6207

$\sigma_p/f_{p0.2}$	Total relaxation	t [min]	Dvp. of relax.
(1)	(2)	(3)	(4)
0.6	0.02	2	0.20
0.7	0.04	3	0.23
0.8	0.07	5	0.26
0.9	0.10	10	0.31
1.0	0.14	1 year	1.00

Column (2) “total relaxation” of the table contains the total loss of stress due to relaxation at time infinity as the multiple of the acting stress, i.e. the loss of stress at t_{∞} which is the total relaxation capacity depending on the applied stress. Columns (4) and (5) named “development of relaxation” contain the loss of stress due to relaxation depending on time expressed as a multiple of the total relaxation at time infinity (500 000 hours), i.e. the time-development of the stress loss over time. With respect to the previous formula, the values in columns (4) and (5) of Table 4-4 (a) are calculated as

$$\frac{\Delta\sigma_{pr}(t)}{\Delta\sigma_{pr}(\infty)} = \left(\frac{t}{500\,000} \right)^{0.75(1-\mu)} \quad (4.38)$$

For comparison the values in column (4) of Table 4-4 (b) are taken from Table 15 of ČSN 73 6207, 1993.

Let us assume that the stress introduced in the prestressing reinforcement by the jack during stressing $\sigma_{p0} = 1\,440$ MPa is kept for 5 minutes. The calculation is performed in Table 4-5. The total relaxation can be determined from Table 4-4 by means of interpolation for the actually acting stress. The total capacity of relaxation at the stress of 1 440 MPa is thus $\Delta\sigma_{pr}^{capacity} = -87.570$ MPa for EN and -167.718 MPa for ČSN (the negative sign means the loss of stress). The column named “development of relaxation” contains the loss of stress due to relaxation depending on time expressed as a multiple of the introduced stress.

Table 4-5 Calculation of relaxation correction through keeping the stress

(a) EN 1992-1-1, 2004

σ_{p0} [MPa]	σ_p/f_{pk}	Total relaxation	t [min]	Development of relaxation
1440	0.8000	0.0608	5	0.00585
	$\Delta\sigma_{pr}^{capacity}$	-87.570	$\Delta\sigma_{pr}^{cor}$	-8.424
			$\Delta\sigma_{pr}^{remaining}$	-79.144

(b) ČSN 73 6207

σ_{p0} [MPa]	$\sigma_p/f_{p0.2}$	Total relaxation	t [min]	Development of relaxation
1440	0.9412	0.1165	5	0.0303
	$\Delta\sigma_{pr}^{capacity}$	-167.718	$\Delta\sigma_{pr}^{cor}$	-43.607
			$\Delta\sigma_{pr}^{remaining}$	-124.111

The value of this loss can be determined by multiplication of the total relaxation e.g. for EN 0.0608 by the value 0.0962 that corresponds to keeping the stress for 5 minutes, see Table 4-4. The relaxation equivalent to the creep that occurred during the time over which the stress was kept constant can then be calculated as $\Delta\sigma_{pr}^{cor} = -0.00585 * 1\,440 = -8.424$ MPa. This is the value by which the total capacity of relaxation is reduced in subsequent calculations.

As we can see from the table, the value of the loss calculated in the same way according to the referenced ČSN is significantly higher $\Delta\sigma_{pr}^{cor} = -43.607$ MPa, which is caused by the facts that in EN 1992-1-1 (i) the development of the relaxation loss over time depends on the relative tendon stress and it is slower for stress level 0.7 to 0.8 f_{pk} , and (ii) the relative stress is related to the characteristic tensile strength and not to the characteristic 0.2% proof-stress, which increases the total relaxation capacity.

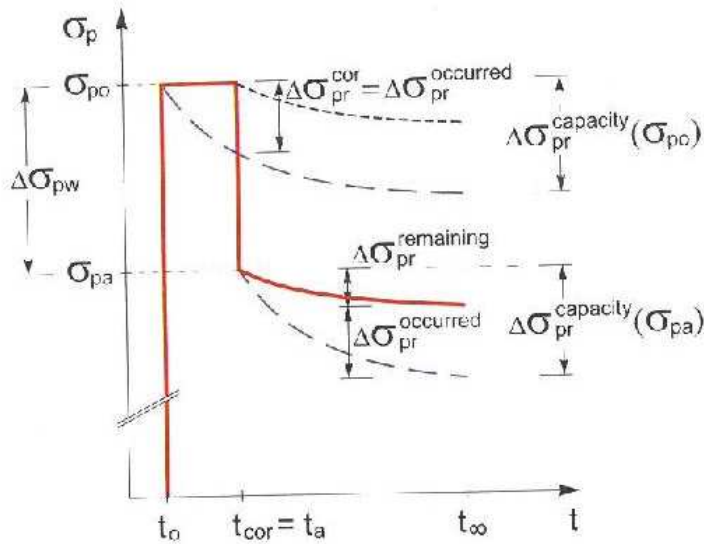


Fig. 4-14 Calculation of the loss due to relaxation of prestressing reinforcement

After the phase of keeping the stress is over, the prestressing reinforcement is anchored. This causes a reduction of the stress in the prestressing reinforcement by anchorage set loss $\Delta\sigma_{pw}$, see Fig. 4-14. However, a reduced capacity of relaxation $\Delta\sigma_{pr}^{capacity}(\sigma_{pa})$ corresponds to the actual stress σ_{pa} and this capacity of relaxation must be further reduced by the relaxation that already took place during the phase when the stress was kept constant, i.e.

$\Delta\sigma_{pr}^{occurred} = \Delta\sigma_{pr}^{cor}$. The calculation is shown in Table 4-6, in which formula (4.37) was directly applied to the

calculation without the use of the relaxation tables.

Table 4-6 Calculation of relaxation at 365 days, EN 1992-1-1

σ_{pa} [MPa]	ρ_{1000} [%]	σ_p/f_{pk}	Total relaxation	t_{5min} [hours]	$t_{365days}$ [hours]	Development of relaxation
1280	2.5	0.7111	0.04099	0.0833	8760	0.38233
$\Delta\sigma_{pr}^{occurred}$		$\Delta\sigma_{pr}^{capacity}$	-52.461		$\Delta\sigma_{pr}^{365}$	-16.836
-8.426		$\Delta\sigma_{pr}^{remaining}$	-44.035			

This calculation can be further refined according to Annex D of EN 1992-1-1, 2004, which requires that the relaxation already occurred is added to the currently acting stress when calculating the capacity of relaxation. Therefore $\Delta\sigma_{pr}^{capacity}(\sigma_{pa} + \Sigma \Delta\sigma_{pr})$ corresponds to the actual stress $\sigma_{pa} + \Sigma \Delta\sigma_{pr}$. This method was used for the comparative calculations in Table 4-7. We experience again a significant difference in the calculated values. Therefore, it is strongly recommended to use specific relaxation tables provided by the producer of the prestressing reinforcement.

Table 4-7 Calculation of relaxation at 365 days according to Annex D of EN 1992-1-1

EN 1992-1-1	σ_{pa} [MPa]	$\sigma_{pa} + \Delta\sigma_{pr}$ [MPa]	μ	estimated t_c [hours]	t_{365} [hours]	$\Delta t = t_{365} - t_a$ [hours]	$t_c + \Delta t$ [hours]	$\Delta\sigma_{pr}^{365}$ [MPa]
	1280	1288.426	0.7158	82.66	8760	8759.917	8842.577	-14.387
ČSN 73 6207	σ_{pa} [MPa]	$\sigma_{pa} + \Delta\sigma_{pr}$ [MPa]	σ_p/f_{02}	estimated t_c [days]	t_{365} [days]	$\Delta t = t_{365} - t_a$ [days]	$t_c + \Delta t$ [days]	$\Delta\sigma_{pr}^{365}$ [MPa]
	1280	1323.607	0.8651	0.01706	365	364.997	365.014	-74.897

The general rule is that after each change of the stress in the prestressing reinforcement, the capacity of relaxation should be re-calculated and the relaxation that already occurred should be deducted. In Fig. 4-15 for example, the presented calculation would have to be modified, as after the stress was kept constant, *short-term relaxation* $\Delta\sigma_{pr}^1$ took place between times t_{cor} and t_a , and it must be added to the relaxation that took place during the period of keeping the stress constant, and only then the remaining capacity of relaxation $\Delta\sigma_{pr}^2$, or the *long-term relaxation* that occurred between the anchoring time and the selected time $t > t_a$, can be calculated.

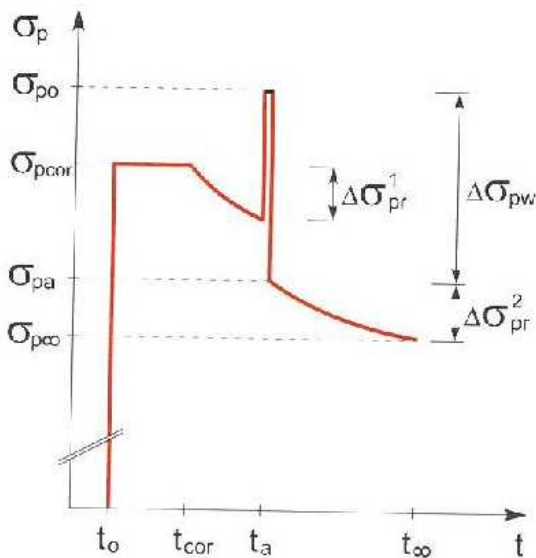


Fig. 4-15 General stress history with a partial reduction of anchorage set loss

The calculation of the long-term relaxation (belongs to long-term losses) must be carried out with regard to the procedure of stressing, anchoring and long-term behaviour of the civil engineering structure (reduction of the stress in the reinforcement due to creep and shrinkage). The calculation is based on the principles stated above. First, the total (capacity of) relaxation that depends on the stress level in the reinforcement in given sections of the element or structure is calculated. Then, the relaxation that took place in the previous steps is deducted and the remaining capacity of relaxation is used to determine the long-term relaxation over the given time interval. In this type of calculation, we accept the simplification that changes in the stress in the prestressing reinforcement that occur during a given time interval do not influence the magnitude of relaxation over this interval.

Generally, what must be determined is the prestressing force and thus also the relaxation at time instants (i) t_a immediately after anchoring, or after the transfer of the prestressing into the concrete, (ii) t_g after the introduction of all permanent loads including prestressing, (iii) t_q when the variable load has been applied and (iv) at the time close to infinity t_∞ .